Solopka, I. O.

On the Andrews-Curtis hypothesis. (Russian) Zbl 0601.20035

Monogenic functions and mappings, Collect. sci. Works, Kiev 1982, 52-58 (1982).

[For the entire collection see Zbl 0506.00009.]

Nielsen's theorem that the automorphism group $\mathrm{Aut}(F_k)$ of a free group of rank k can be finitely generated by elementary automorphisms corresponding to Nielsen operations can be restated as follows [see *J. J. Andrews* and *M. L. Curtis*, Proc. Am. Math. Soc. 16, 192-195 (1965; Zbl 0131.38301)]: if $X =< x_1, ..., x_k >$ and $Y =< y_1, ..., y_k >$ are free bases of F_k, then there is a finite sequence of operations of types (1)-(3) that will change X into Y.

(1) Permute the elements in the given set X;

(2) Replace x_1 by its inverse x_1^{-1};

(3) Replace x_1 by the product x_1x_2;

(4) Replace x_1 by any conjugate $g^{-1}x_1g$, where $g \in F_k$.

Conjecture. [Andrews and Curtis, op. cit.]. If $R =< r_1, ..., r_k >\subseteq F_k$ has normal closure equal to F_k (i.e., if $< X|R >$ is a presentation of the trivial group), then R may be changed into X by a finite sequence of operations of types (1)-(4).

The author of the paper under review establishes the following partial result for "short" basis elements. Theorem. Let $U = (u_1, ..., u_m) \subseteq F_k$ be a subset of F_k whose normal closure is the entire group F_k. If $r \leq 4$ and if $\alpha = \prod_{i=1}^r g_i^{-1} v_i g_i$, where $g_i \in F_k$, $v_i \in U^{+1}$, then U can be transformed by operations (1)-(4) into $U' = (\alpha, u_1, ..., \tilde{u}_j, ..., u_m)$- with a u_j omitted.

Although a general theorem is not proved, the author offers the following main result. Recall that an element of a free group F is primitive if the quotient by its normal closure is a free group of rank rank(F)-1.

Theorem 2. A necessary and sufficient condition for the generalized Andrews-Curtis hypothesis to hold is that there exists a primitive element is some subgroup of F_k generated by some set obtained from $R =< r_1, ..., r_m >$ by operations of types (1)-(4). - The generalized Andrews-Curtis hypothesis allows any number of normal generators (necessarily $m \geq k$) and requires that R be transformed into (X,1,1,...,1), where X is any basis of F_k (with k elements).

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Nielsen's theorem; automorphism group; free group; elementary automorphisms; free bases; normal closure; presentation; generalized Andrews-Curtis hypothesis; primitive element; normal generators