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The multiplicative anomaly for determinants revisited; locality. (English) Zbl 1246.47012
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The paper deals with the multiplicative anomaly of ζ -determinants. The authors review the construction and properties of logarithms of elliptic operators and prove that the expression $L(A, B) := \log(AB) - \log(A) - \log(B)$ is a finite sum of commutators of zero order classical pseudodifferential operators. Basic properties of weighted traces are also reviewed and it is proved that the canonical and weighted traces as well as the noncommutative residue commute with differentiation on differentiable families of operators with constant order.

Moreover, for two admissible pseudodifferential operators A and B (acting on smooth sections of a certain vector bundle and) having non negative orders, the authors prove that the weighted trace of $L(A, B)$ is a local expression as a finite sum of noncommutative residues, which only depends on the first n homogeneous components of the symbols of A and B . In addition, an explicit local expression for the weighted traces of $L(A, B)$ is derived. This allows the authors to derive an explicit local formula for the multiplicative anomaly of ζ -determinants.

Throughout the text, detailed interconnections are drawn with several known results.

Reviewer: Luis Filipe Pinheiro de Castro (Aveiro)

MSC:

- 47G30 Pseudodifferential operators
- 11M36 Selberg zeta functions and regularized determinants; applications to spectral theory, Dirichlet series, Eisenstein series, etc. (explicit formulas)
- 35R01 PDEs on manifolds
- 58J40 Pseudodifferential and Fourier integral operators on manifolds

Keywords:

pseudodifferential operator; noncommutative residue; canonical and weighted traces; zeta and weighted determinants; multiplicative anomaly

Full Text: [Euclid](#) [arXiv](#)

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