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**Short time uniqueness results for solutions of nonlocal and non-monotone geometric equations.** (English) Zbl 1246.35013

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The authors describe a method to show short time uniqueness results for viscosity solutions of general nonlocal and non-monotone second order geometric equations arising in front propagation problems. Their methods is based on some lower gradient bounds for the solution. This results are then used to obtain short time uniqueness results for the initial value problems for dislocation type equations, asymptotic equations of a FitzHugh-Nagumo type system and equations depending on the Lebesgue measure of the fronts.

Reviewer: [Shu-Yu Hsu \(Min-hsiung\)](#)

**MSC:**

- [35A02](#) Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness
- [35K15](#) Initial value problems for second-order parabolic equations
- [49L25](#) Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games
- [45K05](#) Integro-partial differential equations
- [53C44](#) Geometric evolution equations (mean curvature flow, Ricci flow, etc.) (MSC2010)
- [35D40](#) Viscosity solutions to PDEs
- [35R09](#) Integral partial differential equations

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**Keywords:**

front propagation problems; lower gradient bounds; dislocation type equation; FitzHugh-Nagumo type system

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**References:**

- [1] Alvarez O., Cardaliaguet P., Monneau R.: Existence and uniqueness for dislocation dynamics with nonnegative velocity. *Interfaces Free Bound.* 7(4), 415–434 (2005) · [Zbl 1099.35148](#) · [doi:10.4171/IFB/131](#)
- [2] Alvarez O., Hoch P., Le Bouar Y., Monneau R.: Dislocation dynamics: short-time existence and uniqueness of the solution. *Arch. Ration. Mech. Anal.* 181(3), 449–504 (2006) · [Zbl 1158.74335](#) · [doi:10.1007/s00205-006-0418-5](#)
- [3] Aubin J.-P., Frankowska H.: *Set-valued Analysis*. Modern Birkhäuser Classics. Birkhäuser Boston Inc., Boston (2009)
- [4] Barles G.: A new stability result for viscosity solutions of nonlinear parabolic equations with weak convergence in time. *C. R. Math. Acad. Sci. Paris* 343(3), 173–178 (2006) · [Zbl 1102.35014](#) · [doi:10.1016/j.crma.2006.06.022](#)
- [5] Barles G., Biton S., Ley O.: A geometrical approach to the study of unbounded solutions of quasilinear parabolic equations. *Arch. Ration. Mech. Anal.* 162(4), 287–325 (2002) · [Zbl 1052.35084](#) · [doi:10.1007/s002050200188](#)
- [6] Barles G., Cardaliaguet P., Ley O., Monneau R.: Global existence results and uniqueness for dislocation equations. *SIAM J. Math. Anal.* 40(1), 44–69 (2008) · [Zbl 1158.49029](#) · [doi:10.1137/070682083](#)
- [7] Barles G., Cardaliaguet P., Ley O., Monteillet A.: Uniqueness results for nonlocal Hamilton-Jacobi equations. *J. Funct. Anal.* 257, 1261–1287 (2009) · [Zbl 1169.49028](#) · [doi:10.1016/j.jfa.2009.04.014](#)
- [8] Barles G., Cardaliaguet P., Ley O., Monteillet A.: Existence of weak solutions for general nonlocal and nonlinear second-order parabolic equations. *Nonlinear Anal. TMA.* 71, 2801–2810 (2009) · [Zbl 1166.49027](#) · [doi:10.1016/j.na.2009.01.156](#)
- [9] Barles G., Jakobsen E.R.: Error bounds for monotone approximation schemes for parabolic Hamilton- Jacobi-Bellman equations. *Math. Comput.* 76(260), 1861–1893 (2007) · [Zbl 1123.65096](#) · [doi:10.1090/S0025-5718-07-02000-5](#)
- [10] Barles G., Ley O.: Nonlocal first-order Hamilton-Jacobi equations modelling dislocations dynamics. *Comm. Partial Differ. Equ.* 31(7–9), 1191–1208 (2006) · [Zbl 1109.35026](#) · [doi:10.1080/03605300500361446](#)
- [11] Barles G., Soner H.M., Souganidis P.E.: Front propagation and phase field theory. *SIAM J. Control Optim.* 31(2), 439–469

- (1993) · [Zbl 0785.35049](#) · [doi:10.1137/0331021](#)
- [12] Barles G., Souganidis P.E.: A new approach to front propagation problems: theory and applications. *Arch. Rational Mech. Anal.* 141(3), 237–296 (1998) · [Zbl 0904.35034](#) · [doi:10.1007/s002050050077](#)
- [13] Bellettini G., Paolini M.: Two examples of fattening for the curvature flow with a driving force. *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Mat. Appl.* 5(9), 229–236 (1994) · [Zbl 0826.35051](#)
- [14] Biton S., Cardaliaguet P., Ley O.: Non fattening condition for the generalized evolution by mean curvature and applications. *Interfaces Free Bound.* 10, 1–14 (2008) · [Zbl 1154.53041](#) · [doi:10.4171/IFB/177](#)
- [15] Bourgoing M.: Viscosity solutions of fully nonlinear second order parabolic equations with L 1 dependence in time and Neumann boundary conditions. *Discrete Contin. Dyn. Syst.* 21(3), 763–800 (2008) · [Zbl 1152.35411](#) · [doi:10.3934/dcds.2008.21.763](#)
- [16] Bourgoing M.: Viscosity solutions of fully nonlinear second order parabolic equations with L 1 dependence in time and Neumann boundary conditions. Existence and applications to the level-set approach. *Discrete Contin. Dyn. Syst.* 21(4), 1047–1069 (2008) · [Zbl 1165.35401](#) · [doi:10.3934/dcds.2008.21.1047](#)
- [17] Cardaliaguet P.: On front propagation problems with nonlocal terms. *Adv. Differ. Equ.* 5(1–3), 213–268 (2000) · [Zbl 1029.53081](#)
- [18] Cardaliaguet P., Pasquignon D.: On the approximation of front propagation problems with nonlocal terms. *Math. Model. Numer. Anal.* 35(3), 437–462 (2001) · [Zbl 0992.65097](#) · [doi:10.1051/m2an:2001120](#)
- [19] Chen Y.G., Giga Y., Goto S.: Uniqueness and existence of viscosity solutions of generalized mean curvature flow equations. *J. Differ. Geom.* 33(3), 749–786 (1991) · [Zbl 0696.35087](#)
- [20] Chen X., Hilhorst D., Logak E.: Asymptotic behavior of solutions of an Allen-Cahn equation with a nonlocal term. *Nonlinear Anal.* 28(7), 1283–1298 (1997) · [Zbl 0883.35013](#) · [doi:10.1016/S0362-546X\(97\)82875-1](#)
- [21] Clarke F.H., Ledyaev Yu.S., Stern R.J., Wolenski P.R.: *Nonsmooth Analysis and Control Theory*. Springer-Verlag, New York (1998) · [Zbl 1047.49500](#)
- [22] Crandall M.G., Ishii H., Lions P.-L.: User’s guide to viscosity solutions of second order partial differential equations. *Bull. Am. Math. Soc. (N.S.)* 27(1), 1–67 (1992) · [Zbl 0755.35015](#) · [doi:10.1090/S0273-0979-1992-00266-5](#)
- [23] Da Lio F., Kim C.I., Slepcev D.: Nonlocal front propagation problems in bounded domains with Neumann-type boundary conditions and applications. *Asymptot. Anal.* 37(3–4), 257–292 (2004)
- [24] Evans, L.C., Gariepy, R.F.: *Measure theory and fine properties of functions*. In: *Studies in Advanced Mathematics*. CRC Press, Boca Raton (1992) · [Zbl 0804.28001](#)
- [25] Evans L.C., Spruck J.: Motion of level sets by mean curvature. I. *J. Differ. Geom.* 33(3), 635–681 (1991) · [Zbl 0726.53029](#)
- [26] Foote R.L.: Regularity of the distance function. *Proc. Am. Math. Soc.* 92, 153–155 (1984) · [Zbl 0528.53005](#)
- [27] Forcadel N.: Dislocation dynamics with a mean curvature term: short time existence and uniqueness. *Differ. Integr. Equ.* 21(3–4), 285–304 (2008) · [Zbl 1224.49038](#)
- [28] Forcadel N., Montillet A.: Minimizing movements for dislocation dynamics with a mean curvature term. *ESAIM Control Optim. Calc. Var.* 15(1), 214–244 (2009) · [Zbl 1194.49064](#) · [doi:10.1051/cocv:2008027](#)
- [29] Giga, Y.: *Surface evolution equations. A level set approach*. In: *Monographs in Mathematics*, vol. 99. Birkhäuser Verlag, Basel (2006) · [Zbl 1096.53039](#)
- [30] Giga Y., Goto S., Ishii H.: Global existence of weak solutions for interface equations coupled with diffusion equations. *SIAM J. Math. Anal.* 23(4), 821–835 (1992) · [Zbl 0754.35061](#) · [doi:10.1137/0523043](#)
- [31] Gulliver R., Koo Y.: Sharp growth rate for generalized solutions evolving by mean curvature plus a forcing term. *J. Reine Angew. Math.* 538, 1–24 (2001) · [Zbl 1037.53045](#) · [doi:10.1515/crll.2001.065](#)
- [32] Henrot A., Pierre M.: *Variation et optimisation de formes*. Springer, Berlin (2005) · [Zbl 1098.49001](#)
- [33] Ishii H.: Hamilton-Jacobi equations with discontinuous Hamiltonians on arbitrary open sets. *Bull. Fac. Sci. Eng. Chuo Univ.* 28, 33–77 (1985) · [Zbl 0937.35505](#)
- [34] Jakobsen E.R., Karlsen K.H.: Continuous dependence estimates for viscosity solutions of fully nonlinear degenerate parabolic equations. *J. Differ. Equ.* 183(2), 497–525 (2002) · [Zbl 1086.35061](#) · [doi:10.1006/jdeq.2001.4136](#)
- [35] Jakobsen, E.R., Karlsen, K.H.: Continuous dependence estimates for viscosity solutions of fully nonlinear degenerate elliptic equations. *Electron. J. Differ. Equ.* 39 (2002) · [Zbl 1010.35050](#)
- [36] Koo Y.: A fattening principle for fronts propagating by mean curvature plus a driving force. *Comm. Partial Differ. Equ.* 24(5–6), 1035–1053 (1999) · [Zbl 0935.35035](#) · [doi:10.1080/03605309908821457](#)
- [37] Ley O.: Lower-bound gradient estimates for first-order Hamilton-Jacobi equations and applications to the regularity of propagating fronts. *Adv. Differ. Equ.* 6(5), 547–576 (2001) · [Zbl 1015.35031](#)
- [38] Maz’ya V.G., Poborchii S.V.: *Differentiable Functions on Bad Domains*. World Scientific, River Edge (1997) · [Zbl 0918.46033](#)
- [39] Nunziante D.: Existence and uniqueness of unbounded viscosity solutions of parabolic equations with discontinuous time-dependence. *Nonlinear Anal.* 18(11), 1033–1062 (1992) · [Zbl 0782.35037](#) · [doi:10.1016/0362-546X\(92\)90194-J](#)
- [40] Osher S., Osher S.: Fronts moving with curvature dependent speed: algorithms based on Hamilton-Jacobi equations. *J. Comput. Phys.* 79, 12–49 (1988) · [Zbl 0659.65132](#) · [doi:10.1016/0021-9991\(88\)90002-2](#)
- [41] Rodney D., Le Bouar Y., Finel A.: Phase-field methods and dislocations. *Acta Mater.* 51, 17–30 (2003) · [doi:10.1016/S1359-6454\(01\)00379-2](#)
- [42] Slepčev D.: Approximation schemes for propagation of fronts with nonlocal velocities and Neumann boundary conditions. *Nonlinear Anal.* 52(1), 79–115 (2003) · [Zbl 1028.35068](#) · [doi:10.1016/S0362-546X\(02\)00098-6](#)

- [43] Soravia P., Souganidis P.E.: Phase-field theory for FitzHugh-Nagumo-type systems. *SIAM J. Math. Anal.* 27(5), 1341–1359 (1996) · [Zbl 0863.35047](#) · [doi:10.1137/S0036141094261318](#)
- [44] Srour A.: Nonlocal second-order Hamilton-Jacobi equations arising in tomographic reconstruction. *Nonlinear Anal. TMA* 71, 1746–1762 (2009) · [Zbl 1177.35116](#) · [doi:10.1016/j.na.2008.02.077](#)

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