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Type II Hermite-Padé approximations of generalized hypergeometric series. (English)

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Constr. Approx. 33, No. 3, 289-312 (2011).

The author studies simultaneous Hermite-Padé approximation to generalized hypergeometric and q-hypergeometric series of the form

$$F(t)=\sum_{n=0}^{\infty}\frac{\prod_{k=0}^{n-1}P(k)}{\prod_{k=0}^{n-1}Q(k)}t^n,\ F_q(t)=\sum_{n=0}^{\infty}\frac{\prod_{k=0}^{n-1}P(q^k)}{\prod_{k=0}^{n-1}Q(q^k)}t^n,$$

where P and Q are polynomials.

The main method used to prove the results is based on a paper by *W. Maier* [J. f. M. 156, 93–148 (1927; JFM 53.0340.02)] and a recent modification introduced by the author [J. Math. Soc. Japan 61, No. 1, 291–213 (2009; Zbl 1169.11031)].

The interest in the type of results given, originates from the study of simultaneous approximation in the context of algebraic independence and irrationality in the Theory of Numbers.

After a short introduction there are four sections dedicated to

A. Hypergeometric functions F, with its main result the explicit form of the type II Hermite-Padé approximants in the variable t for the d series $\theta^b F(t), 0\le b\le d-1$ at m points (for $d=1$ the theorem gives the type II simultaneous approximants). NB. $\theta=t\frac{d}{dt}$.

Furthermore, several applications are given (classical hypergeometric seris, exponential series - these are actually ${}_1F_1(1;a;z)$ series - , logarithmic and polylogarithmic series)

B. The remainder series technique (using the modified Maier technique), leading to diagonal Hermite-Padé approximants with a free parameter.

C. q-hypergeometric functions F_q as given above, giving a new proof of a theorem due to *Th. Stihl* [Math. Ann. 268, 21–41 (1984; Zbl 0519.10024)].

D. A new proof (using the modified Stihl-Maier method) for a result due to the author in a previous paper [Zbl 1169.11031].

The paper concludes with an appendix with some results on Stirling numbers (needed for the proofs) and a list of 18 references.

Reviewer: Marcel G. de Bruin (Haarlem)

MSC:

41A21 Padé approximation
41A28 Simultaneous approximation
33C20 Generalized hypergeometric series, ${}_pF_q$
33D99 Basic hypergeometric functions

Keywords:

hypergeometric series; q-hypergeometric series; Padé approximation; simultaneous approximation

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