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Meshfree-enriched simplex elements with strain smoothing for the finite element analysis of compressible and nearly incompressible solids. (English) Zbl 1230.74201

Comput. Methods Appl. Mech. Eng. 200, No. 45-46, 2991-3010 (2011).

Summary: This paper presents a meshfree-enriched finite element formulation for triangular and tetrahedral elements in the analysis of two and three-dimensional compressible and nearly incompressible solids. The new formulation is first established in two-dimensional case by introducing a meshfree approximation into a linear triangular finite element with an enriched node. The interpolation functions of the four-noded triangular element are constructed by the meshfree convex approximations and are completed to a polynomial of degree one. The reference mapping using the constructed interpolation functions is shown to be invertible everywhere in the element and the global element area is proven to be conserved under a standard three-point integration rule. The triangular element formulation is extendable to the tetrahedral element in three-dimensional case. To provide a locking-free analysis for the nearly incompressible materials, an area-weighted strain smoothing is developed in conjunction with the enriched interpolation functions to yield a discrete divergence-free property at the integration point. The resultant element formulation with strain smoothing is shown to pass the patch test. To introduce the smoothed strain into Galerkin formulation, a modified Hu-Washizu variational principle is adopted to formulate the discrete equations. Since the Kronecker-delta property in element interpolation is held along the element boundary using meshfree convex approximation, boundary conditions can be treated in a standard way. Several numerical benchmarks are provided to demonstrate the effectiveness and accuracy of the proposed method.

MSC:

74S05 Finite element methods applied to problems in solid mechanics
74B05 Classical linear elasticity

Keywords:

meshfree; triangular element; finite element; convex; volumetric locking

Full Text: DOI

References:

[1] Andrade Pires, F.M.; de Souza Neto, E.A.; de la Cuesta Padilla, J.L., An assessment of the average nodal volume formulation for the analysis of nearly incompressible solids under finite strains, Int. J. numer. methods engrg., 20, 569-583, (2004) · Zbl 1302.74173

[2] Arnold, D.N.; Brezzi, F.; Franca, L.P., A stable finite element for the Stokes equations, Calcolo, 21, 337-344, (1984) · Zbl 0593.76039

[3] Arroyo, M.; Ortiz, M., Local maximum-entropy approximation schemes: a seamless bridge between finite elements and meshfree methods, Int. J. numer. methods engrg., 65, 2167-2202, (2006) · Zbl 1146.74048

[4] Arunakirinathar, K.; Reddy, B.D., A stable affine-approximate finite element method, SIAM J. numer. anal., 40, 180-197, (2002) · Zbl 1215.74077

[5] Bathe, K.J., Finite element procedures, (1996), Prentice-Hall New Jersey · Zbl 0511.73065

[6] Babuska, I., The finite element method with Lagrangian multipliers, Numer. math., 20, 179-192, (1973) · Zbl 0258.65108

[7] Belytschko, T.; Lu, Y.Y.; Gu, L., Element-free Galerkin methods, Int. J. numer. methods engrg., 37, 229-256, (1994) · Zbl 0796.73077

[8] Bonet, J.; Burton, A.J., A simple average nodal pressure tetrahedral element for incompressible and nearly incompressible dynamic explicit applications, Commun. numer. methods engrg., 14, 437-449, (1998) · Zbl 0906.73060

[9] Brenner, S.; Sung, L.Y., Linear finite element methods for planar linear elasticity, Math. comput., 59, 321-338, (1992) · Zbl 0766.73060

[10] César De Sá, J.M.A.; Natal Jorge, R.M., New enhanced strain elements for incompressible problems, Int. J. numer. methods engrg., 44, 229-248, (1999) · Zbl 0937.74062

[11] Chen, J.S.; Pan, C.; Wu, C.T.; Liu, W.K., Reproducing kernel particle methods for large deformation analysis of non-linear structures, Comput. methods appl. mech. engrg., 139, 195-227, (1996) · Zbl 0918.73330

[12] Chen, J.S.; Wang, H.P., New boundary condition treatments in meshfree computation of contact problems, Comput. methods appl. mech. engrg., 187, 441-468, (2000) · Zbl 0980.74077

[13] Chen, J.S.; Yoon, S.; Wang, H.P.; Liu, W.K., An improved reproducing kernel particle method for nearly incompressible finite elasticity, Comput. methods appl. mech. engrg., 181, 117-145, (2000) · Zbl 0973.74088

[14] Chen, J.S.; Wu, C.T.; Yoon, S.; You, Y., A stabilized conforming nodal integration for Galerkin mesh-free methods, Int. J. numer. methods engrg., 50, 435-466, (2001) · Zbl 1011.74081

[15] Ciarlet, P.G., The finite element method for elliptic problems, (1978), North-Holland Publishing Company Amsterdam · Zbl 0445.73043

[16] Cui, X.; Liu, G.R.; Li, G.Y.; Zhang, G.Y.; Zheng, G., Analysis of plates and shells using an edge-based smoothed finite element method, Comput. mech., 45, 141-156, (2010) · Zbl 1202.74165

[17] De, S.; Bathe, K.J., The method of finite spheres, Comput. mech., 25, 329-345, (2000) · Zbl 0952.65091

[18] De, S.; Bathe, K.J., Displacement/pressure mixed interpolation in the method of finite spheres, Int. J. numer. methods engrg., 51, 275-292, (2001) · Zbl 0995.74081

[19] Djoko, J.K.; Lamichhane, B.P.; Reddy, B.D.; Wohlmuth, B.I., Conditions for equivalence between the hu – washizu and related formulations, and computational behavior in the incompressible limit, Comput. methods appl. mech. engrg., 195, 4161-4178, (2006) · Zbl 1123.74020

[20] Dolbow, J.; Belytschko, T., Volumetric locking in the element free Galerkin method, Int. J. numer. methods engrg., 46, 925-942, (1999) · Zbl 0967.74079

[21] Dolbow, J.; Belytschko, T., Numerical integration of Galerkin weak form in meshfree methods, Comput. mech., 23, 219-230, (1999) · Zbl 0963.74076

[22] Falk, R.S., Nonconforming finite element methods for the equations of linear elasticity, Math. comput., 57, 529-550, (1991) · Zbl 0747.73044

[23] Guo, Y.; Ortiz, M.; Belytschko, T.; Repetto, E.A., Triangular composite finite elements, Int. J. numer. methods engrg., 47, 287-316, (2000) · Zbl 0985.74068

[24] Hansbo, P.; Larson, M.G., Discontinuous Galerkin and the crouzeix – raviart element: application to elasticity, Math. model. numer. anal., 37, 63-72, (2003) · Zbl 1137.65431

[25] Hauret, P.; Kuhl, E.; Ortiz, M., Diamond elements: a finite element/discrete-mechanics approximation scheme with guaranteed optimal convergence in incompressible elasticity, Int. J. numer. methods engrg., 72, 253-294, (2007) · Zbl 1194.74406

[26] Huerta, A.; Fernandez-Mendez, S., Locking in the incompressible limit for the element-free Galerkin method, Int. J. numer. methods engrg., 51, 1361-1383, (2001) · Zbl 1065.74635

[27] Hueck, U.; Schreyer, H.; Wriggers, P., On the incompressible constraint of the 4-node quadrilateral element, Int. J. numer. methods engrg., 38, 3039-3053, (1995) · Zbl 0854.73066

[28] Hughes, T.J.R., The finite element method, (2000), Prentice-Hall Englewood Cliffs, NJ

[29] Krysl, P.; Zhu, B., Locking-free continuum displacement finite elements with nodal integration, Int. J. numer. methods engrg., 76, 1020-1043, (2008) · Zbl 1195.74182

[30] Lamichhane, B.P., Inf-sup stable finite element pairs based on dual meshes and bases for nearly incompressible elasticity, IMA J. numer. anal., 29, 404-420, (2009) · Zbl 1160.74046

[31] Lamichhane, B.P., From the hu – washizu formulation to the average nodal strain formulation, Comput. method appl. mech. engrg., 198, 3957-3961, (2009) · Zbl 1231.74424

[32] Liu, G.R.; Nguyen, T.T.; Dai, K.Y.; Lam, K.Y., Theoretical aspects of the smoothed finite element method (SFEM), Int. J. numer. methods engrg., 71, 902-930, (2007) · Zbl 1194.74432

[33] Liu, G.R.; Zhang, G.Y., Upper bound solution to elasticity problems: a unique property of the linearly conforming point interpolation method (LC-PIM), Int. J. numer. methods engrg., 74, 1128-1161, (2008) · Zbl 1158.74532

[34] Liu, G.R.; Nguyen-Xuan, H.; Nguyen-Thoi, T., A theoretical study on the smoothed FEM method (S-FEM) models: properties, accuracy and convergence rates, Int. J. numer. methods engrg., 84, 1222-1256, (2010) · Zbl 1202.74180

[35] Liu, W.K.; Jun, S.; Li, S.; Adee, J.; Belytschko, T., Reproducing kernel particle methods for structural dynamics, Int. J. numer. methods engrg., 38, 1655-1679, (1995) · Zbl 0840.73078

[36] Lovadina, C.; Auricchio, F., On the enhanced strain technique for elasticity problems, Comput. struct., 81, 777-787, (2003)

[37] Malkus, D.S.; Hughes, T.J.R., Mixed finite element methods-reduced and selective integration techniques: a unification of concepts, Comput. methods appl. mech. engrg., 15, 63-81, (1978) · Zbl 0381.73075

[38] Oden, J.T.; Kikuchi, N.; Song, Y.J., Penalty-finite element methods for the analysis of Stokesian flows, Comput. methods appl. mech. engrg., 31, 297-329, (1982) · Zbl 0478.76041

[39] Ortiz, A.; Puso, M.A.; Sukumar, N., Maximum-entropy meshfree method for compressible and near-incompressible elasticity, Comput. methods appl. mech. engrg., 199, 1859-1871, (2010) · Zbl 1231.74491

[40] Puso, M.A.; Solberg, J., A stabilized nodally integrated tetrahedral, Int. J. numer. methods engrg., 67, 841-867, (2006) · Zbl 1113.74075

[41] Puso, M.A.; Chen, J.S.; Zywicz, E.; Elmer, W., Meshfree and finite element nodal integration methods, Int. J. numer. methods engrg., 74, 416-446, (2008) · Zbl 1159.74456

[42] Reese, S.; Wriggers, P., A stabilization technique to avoid hourglassing in finite elasticity, Int. J. numer. methods engrg., 74, 416-446, (2000) · Zbl 0983.74070

[43] D. Shepard, A two-dimensional interpolation function for irregularly-spaced data, in: Proceedings of ACM National Conference, New York, 1968, pp. 517-524.

[44] Simo, J.C.; Hughes, T., On the variational foundation of assumed strain methods, ASME J. appl. mech., 53, 51-54, (1986) · Zbl 0592.73019

[45] Simo, J.C.; Rifai, M.S., A class of mixed assumed strain methods and the method of incompatible modes, Int. J. numer. methods engrg., 29, 1595-1638, (1990) · Zbl 0724.73222

[46] Stenberg, R., Error analysis of some finite element methods for the Stokes problem, Math. comput., 54, 495-508, (1990) · Zbl 0702.65095

[47] Sukumar, N., Construction of polygonal interpolants: a maximum entropy approach, Int. J. numer. methods engrg., 61, 2159-2181, (2004) · Zbl 1073.65505

[48] Timoshenko, S.P.; Goodier, J.N., Theory of elasticity, (1970), McGraw-Hill New York · Zbl 0266.73008

[49] Vidal, Y.; Villon, P.; Huerta, A., Locking in the incompressible limit: pseudo-divergence-free element free Galerkin, Commun. numer. methods engrg., 19, 725-735, (2003) · Zbl 1112.74545

[50] Wang, H.P.; Wu, C.T.; Guo, Y.; Botkin, M.E., A coupled meshfree/finite element method for automotive crashworthiness simulations, Int. J. impact engrg., 36, 1210-1222, (2009)

[51] Wang, L.H.; Qi, H., A locking-free scheme of nonconforming rectangular finite element for the planar elasticity, J. comput. math., 22, 641-650, (2004) · Zbl 1088.74046

[52] Washizu, K., Variational methods in elasticity and plasticity, (1982), Pergamon Press New York · Zbl 0164.26001

[53] Wilson, E.L.; Taylor, R.L.; Doherty, W.P.; Ghaboussi, J., Incompatible displacement models, (), 43-57

[54] Wu, C.T.; Koishi, M., A meshfree procedure for the microscopic analysis of particle-reinforced rubber compounds, Interact. multiscale mech., 2, 47-169, (2009)

[55] Wu, C.T.; Park, C.K.; Chen, J.S., A generalized meshfree approximation for the meshfree analysis of solids, Int. J. numer. methods engrg., 85, 693-722, (2011) · Zbl 1217.74150

[56] C.T. Wu, W. Hu, J.S. Chen, Meshfree-enriched finite element methods for the compressible and near-incompressible elasticity, Int. J. Numer. Methods Engrg., submitted for publication. · Zbl 1242.74174

[57] Zhao, X.; Liu, G.R.; Dai, K.Y.; Zhong, Z.H.; Li, G.Y.; Han, X., A linearly conforming radial point interpolation method (LC-RPIM) for shells, Comput. mech., 43, 403-413, (2009) · Zbl 1162.74512

[58] Zienkiewicz, O.C.; Taylor, R.L., The finite element method, (1987), McGraw-Hill London · Zbl 0991.74002