Cheung, Eric C. K.

On a class of stochastic models with two-sided jumps. (English) Zbl 1235.60126
Queueing Syst. 69, No. 1, 1-28 (2011).

The paper studies the Gerber-Shiu function in a stochastic model involving two-sided jumps and a continuous downward drift. With arbitrary distributions of jump sizes and inter-arrival times, the general structure of the Gerber-Shiu function is studied via an underlying ladder height structure and the use of defective renewal equations. Applications of the Gerber-Shiu function are illustrated in finding (i) the Laplace transforms of the time of the ruin, the time of recovery and the duration of first negative surplus in the ruin context; (ii) the joint Laplace transforms of the busy period and the subsequent idle period in the queueing context; and (iii) the expected total discounted reward for a continuous payment stream payable during idle periods in a queue.

Reviewer: Oleg K. Zakusilo (Kyïv)

MSC:

60K25 Queueing theory (aspects of probability theory)
60K15 Markov renewal processes, semi-Markov processes
90B22 Queues and service in operations research
91B30 Risk theory, insurance (MSC2010)
60J75 Jump processes (MSC2010)

Keywords:

dual risk model; two-sided jumps; $GI/G/1$ queue; negative customers; Gerber-Shiu function; defective renewal equation; time of ruin; time of recovery; busy period; idle period

Full Text: DOI

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