Xue, Jungong; **Alfa, Attahiru S.**

Geometric tail of queue length of low-priority customers in a nonpreemptive priority MAP/PH/1 queue. (English) Zbl 1238.60106

Queueing Syst. 69, No. 1, 45-76 (2011).

The authors earlier studied a discrete-time BMAP/PH/1 queue with preemptive service discipline [Stoch. Models 21, No. 2–3, 799–820 (2005; Zbl 1069.60085)]. In this paper, they study the geometric decay of the tail probability of low-priority customers of a priority MAP/PH/1 queue with non-preemptive service discipline. They use a quasi birth and death (QBD) process to describe a queue with the queue length of high-priority customers playing the role of *level* and the queue length of low-priority customers together with the phases of arrival and service processes of both classes describing a *phase* in each level. This treatment make the G-matrix and R-matrix of the QBD block upper triangular with identical blocks on each diagonal. They obtain a generating function equation for the stationary distribution of the queue length of low-priority customers. They also derive a sufficient condition for geometric decay. Numerical methods are presented.

Reviewer: P. R. Parthasarathy (Chennai)

MSC:

60K25 Queueing theory (aspects of probability theory)
90B22 Queues and service in operations research

Keywords:

priority queue; tail probability; Markovian arrival process; phase-type distribution; decay rate

Full Text: DOI

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