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**Transient heat conduction in a functionally graded strip in contact with well stirred fluid with an outside heat source.** (English) [Zbl 1231.80036](#)

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Summary: A transient heat conduction model is established for a functionally graded strip which is in contact with a well stirred fluid. The fluid receives heat from an outside source. Exact solutions for the temperature distribution for both the functionally graded strip and the fluid are found using the Laplace transform and the inverse Laplace theorem. An iterative procedure is proposed to find the roots of the transcendental equation. Numerical results for the temperature are obtained and the computational accuracy in terms of the roots is proven. The effects of the graded parameter and the heat transfer parameter on the temperature distribution of both the functionally graded strip and the fluid are discussed, which is helpful for a design optimization of the graded material. In particular, the effect of the thickness of the functionally graded strip on the temperature distribution is analyzed, which provides practical references for a geometrical optimization of the structure.

**MSC:**

80A20 Heat and mass transfer, heat flow (MSC2010)  
44A10 Laplace transform  
33C10 Bessel and Airy functions, cylinder functions,  ${}_0F_1$   
65H05 Numerical computation of solutions to single equations

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**Keywords:**

functionally graded materials; transient heat conduction; transcendent equation; temperature distribution

**Full Text:** [DOI](#)

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