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Efficient Chebyshev spectral methods for solving multi-term fractional orders differential equations. (English) Zbl 1228.65126

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Summary: We state and prove a new formula expressing explicitly the derivatives of shifted Chebyshev polynomials of any degree and for any fractional-order in terms of shifted Chebyshev polynomials themselves. We develop also a direct solution technique for solving the linear multi-order fractional differential equations (FDEs) with constant coefficients using a spectral tau method. The spatial approximation with its fractional-order derivatives (described in the Caputo sense) are based on shifted Chebyshev polynomials $T_{L,n}(x)$ with $x \in (0,L), L > 0$ and n is the polynomial degree. We presented a shifted Chebyshev collocation method with shifted Chebyshev-Gauss points used as collocation nodes for solving nonlinear multi-order fractional initial value problems. Several numerical examples are considered aiming to demonstrate the validity and applicability of the proposed techniques and to compare with the existing results.

MSC:

65L60 Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations
34A08 Fractional ordinary differential equations and fractional differential inclusions
26A33 Fractional derivatives and integrals
45J05 Integro-ordinary differential equations

Keywords:
multi-term fractional differential equations; nonlinear fractional differential equations; tau method; collocation method; shifted Chebyshev polynomials; Gauss quadrature

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