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Description and exact maximum and minimum values of the remainder in the problem of the distribution of fractional parts. (English. Russian original) [Zbl 1303.11078](#)

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The paper deals with the distribution of the function $f(x) = \langle \alpha x \rangle$ where $\langle x \rangle$ is the fractional part of the x . The aim of the paper is to investigate how much can be the deviation of this distribution from the uniform one. Let

$$r(\alpha, n, I) = \#\{i : 0 \leq i < n, \langle i\alpha \rangle \in I\} - n|I|$$

$$r^+(\alpha) = \sup_n r(\alpha, n, I), \quad r^-(\alpha) = \inf_n r(\alpha, n, I).$$

The authors consider the intervals of the form $I = [\delta; \delta + \langle m\alpha \rangle]$ and such that $|r^\pm(\alpha)| < \infty$. Using Hecke's formula for $r(\alpha, n, I)$ and investigating different properties of piecewise linear functions, the authors obtain new formula for $r^\pm(\alpha)$. As a corollary they prove that $r^\pm(\alpha)$ can be computed in $O(m)$ operations.

Reviewer: [Dmitriy Frolenkov \(Moscow\)](#)

MSC:

11J71 Distribution modulo one
11K38 Irregularities of distribution, discrepancy
11K31 Special sequences

Cited in **6** Documents

Keywords:

distribution of fractional parts; extreme values of the remainder; piecewise linear functions

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