Like in the paper of *R. M. Solovay* in Isr. J. Math. 25, 287-304 (1976; Zbl 0352.02019) an interpretation * is an assignment of arithmetic sentences to propositional modal formulas such that * preserves logical connectives and $(\Box Q)^*$ is $\Pr(\ulcorner Q^* \urcorner)$ where $\Pr(\cdot)$ is a standard arithmetical formula of provability in PA. The modal representation g(T) of a theory T is the set of propositional formulas Q such that $T \vdash Q^*$ for all *.

So g(T) is the collection of all universal laws of formal provability $\Pr(\cdot)$ which can be deduced by means of theory T. The logics of provability are the values of $g(\cdot)$; and let LP be the class of all logics of provability. The cardinality of LP is $2^{\aleph_0}$ [the author, Semiotika Inf. 14, 115-133 (1980; Zbl 0463.03006)]. The least logic of provability is $GL = K + \Box(\Box p \to p) \to \Box p$ [*R. M. Solovay*, op. cit.].

The paper studies LP and the class L of all extensions of GL. An infinite set of new logics of provability is found. The classification of L is developed on the basis of the notion of trace of a modal formula which is the set of all possible heights of Kripke countermodels for the formula. According to this classification the majority of logics in L are not logics of provability. The decidability of $GL_\omega = GL + \{\neg\Box^n \bot \mid n \in \omega\}$ and some other logics of provability is shown. The problem of the entire description of LP is reduced to the problem of description of all logics of provability from the interval between GL and S (Solovay's logic of all true laws of provability).

MSC:

03B45	Modal logic (including the logic of norms)
03F07	Structure of proofs
03F30	First-order arithmetic and fragments
03F50	Metamathematics of constructive systems
03B25	Decidability of theories and sets of sentences

Cited in **4** Reviews
Cited in **5** Documents

Keywords:

Kripke models; formal provability; logics of provability; extensions of GL; Solovay's logic of all true laws of provability