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The universal Cannon-Thurston map and the boundary of the curve complex. (English)

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The original Cannon-Thurston map is a quotient map from the boundary $\partial\mathbb{H}$ of the hyperbolic plane onto the limit set of a Kleinian group Γ. It was constructed by *J. W. Cannon* and *W. P. Thurston* in their paper ["Group invariant Peano curves", Geom. Topol. 11, 1315–1355 (2007; Zbl 1136.57009)] for the fiber subgroup of the fundamental group of a closed hyperbolic 3-manifold fibering over the circle. It was later on extended by *Y. N. Minsky* in his paper ["Teichmüller geodesics and ends of hyperbolic 3-manifolds", Topology 32, No. 3, 625–647 (1993; Zbl 0793.58010)] and by *M. Mj* in the preprint ["Ending laminations and Cannon-Thurston maps", arXiv:math/0701725 (2007)]. In this quotient map, distinct points are identified if and only if they are ideal points of a leaf of an ending lamination for Γ. Other kinds of "Cannon-Thurston maps" were constructed later on, and they are mentioned in the introduction of the paper under review.

In this paper, the authors construct a map they call the *Universal Cannon-Thurston map*. For this, they consider a closed hyperbolic surface S of genus ≥ 2 with a distinguished point $z \in S$. The curve complexes of S and (S, z) are denoted respectively by $\mathcal{C}(S)$ and $\mathcal{C}(S, z)$. The fundamental group $\pi_1(S)$ acts on $\mathcal{C}(S, z)$ via the inclusion into the mapping class group of (S, z) and this action gives rise to map

$$\Phi : \mathcal{C}(S) \times \mathbb{H} \to \mathcal{C}(S, z)$$

which leads, by restriction and for any given vertex v of $\mathcal{S}$, to a map

$$\Phi_v : \mathbb{H} \to \mathcal{C}(S, z).$$

Here the authors show that if $r \subset \mathbb{H}$ is a geodesic ray that eventually lies in the preimage of some proper essential subsurface of S then $\Phi_v(r) \subset \mathcal{C}(S, z)$ has finite diamater. The remaining rays define a subset $\mathbb{A} \subset \partial\mathbb{H}$ which is of full measure. The authors then show that this map Φ_v has a unique continuous $\pi_1(S)$-equivariant extension

$$\overline{\Phi}_v : \mathbb{H} \cup \mathbb{A} \to \overline{\mathcal{C}}(S, z)$$

and that the map $\partial\Phi = \overline{\Phi}_v|_{\mathbb{A}}$ does not depend on v and that it is a quotient map onto the Gromov boundary $\partial\mathcal{C}(S, z)$ of the curve complex. Furthermore, they show that for given distinct points x and y in $\mathbb{A}$, one has $\partial\Phi(x) = \partial\Phi(y)$ if and only if x and y are ideal endpoints of a leaf (or ideal vertices of a complementary polygon) of the lift of any ending lamination on S. The last property is the one that makes the map $\partial\Phi$ universal. The authors also prove that the quotient map

$$\partial\Phi : \mathbb{A} \to \partial\mathcal{C}(S, z)$$

is equivariant with respect to the action of the mapping class group of (S, z). Finally, they prove that the Gromov boundary $\partial\mathcal{C}(S, z)$ is path-connected and locally path-connected. This strengthens a result in [*C. J. Leininger* and *S. Schleimer*, "Connectivity of the space of ending laminations", Duke Math. J. 150, No. 3, 533–575 (2009; Zbl 1190.57013)] in a special case.

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MSC:

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57M50 General geometric structures on low-dimensional manifolds

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