Berinde, Vasile

Generalized coupled fixed point theorems for mixed monotone mappings in partially ordered metric spaces. (English) Zbl 1235.54024

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Let X be a complete metric space with metric d, which is partially ordered. A mapping $F : X \times X \to X$ is called mixed monotone if $F(x, y)$ is monotone nondecreasing in x and monotone nonincreasing in y. A pair $(\overline{x}, \overline{y}) \in X \times X$ is called a coupled fixed point of F if $F(\overline{x}, \overline{y}) = \overline{x}$, $F(\overline{y}, \overline{x}) = \overline{y}$. The main result of the paper is the following theorem.

Theorem. Let X be a partialy ordered complete metric space, let $F : X \times X \to X$ be mixed monotone and such that

(i) There is a constant $k \in [0, 1)$ such that for each $x \geq u$, $y \leq v$

$$d(F(x, y), F(u, v)) + d(F(y, x), F(v, u)) \leq k[d(x, u) + d(y, v)].$$

(ii) There exist $x_0, y_0 \in X$ with

$$x_0 \leq F(x_0, y_0) \quad \text{and} \quad y_0 \leq F(y_0, x_0)$$

or

$$x_0 \geq F(x_0, y_0) \quad \text{and} \quad y_0 \leq F(y_0, x_0).$$

Then F has a coupled fixed point $(\overline{x}, \overline{y})$.

The author also gives conditions under which there exists a unique coupled fixed point. Finally, he applies this theorems to the periodic boundary value problem

$$u' = h(t, u), \quad t \in (0, T), \quad u(0) = u(T)$$

with $h(t, u) = f(t, u) + g(t, u)$.

Reviewer: Klaus R. Schneider (Berlin)

MSC:

54H25	Fixed-point and coincidence theorems (topological aspects)
54E50	Complete metric spaces
54F05	Linearly ordered topological spaces, generalized ordered spaces, and partially ordered spaces
34B15	Nonlinear boundary value problems for ordinary differential equations

Keywords:

metric space; mixed monotone operator; contractive condition; coupled fixed point; periodic boundary value problem

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