Li, Xiaoli; **Su, Jiabao**; **Tian, Rushun**

Multiple periodic solutions of the second order Hamiltonian systems with superlinear terms.

(English) Zbl 1237.37042
J. Math. Anal. Appl. 385, No. 1, 1-11 (2012).

This paper is concerned with the multiplicity of 2π-periodic solutions of the second order Hamiltonian system

$$-\ddot{x} - A(t)x = \lambda x + V'_x(t,x),$$

where $\lambda \in \mathbb{R}$, $A(t)$ is a continuous, 2π-periodic, symmetric matrix-valued function, and the potential $V(t,x)$ is C^2 with 2π-periodicity in t, such that

$$V(t,0) = V'_x(t,0) = V''_x(t,0) = 0$$

and

$$0 < \theta V(t,x) \leq (V'_x(t,x), x)$$

for all $t \in [0,2\pi]$ and $|x|$ sufficiently large. The authors separately impose the following additional hypotheses:

(i) $V''_x(t,x) > 0$ for $|x| > 0$ small and $t \in [0,2\pi]$.

(ii) $V''_x(t,x) < 0$ for $|x| > 0$ small and $t \in [0,2\pi]$.

(iii) $V(t,x) \leq 0$ for $|x| > 0$ small and $t \in [0,2\pi]$.

It is shown, under the assumptions (i), (ii) and (iii), that for any fixed positive integer k there is some $\delta > 0$ such that if

$$\sup_{(t,x)\in[0,2\pi]\times\mathbb{R}^N} V^-(t,x) \leq \delta,$$

then the following properties respectively hold true:

(a) if $\lambda \in (\lambda_k - \delta, \lambda_k)$, there exist at least three distinct nontrivial 2π-periodic solutions;

(b) if $\lambda \in (\lambda_k, \lambda_k + \delta)$, the above problem has at least three distinct nontrivial solutions;

(c) if $\lambda \in (\lambda_k - \delta, \lambda_k]$, the above problem has at least two nontrivial solutions.

The proofs depend on a careful analysis of critical groups, and the solutions are constructed by a combination of bifurcation arguments, topological linking and Morse theory.

Reviewer: Chun-Lei Tang (Chongqing)

MSC:

37J45 Periodic, homoclinic and heteroclinic orbits; variational methods, degree-theoretic methods (MSC2010)
34C25 Periodic solutions to ordinary differential equations
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces

Keywords:

Hamiltonian systems; homological linking; Morse theory; periodic solutions; bifurcation method

Full Text: DOI

References:

[1] Bartsch, T.; Li, S.J., Critical point theory for asymptotically quadratic functionals and applications to problems with resonance, Nonlinear anal., 28, 419-441, (1997) · Zbl 0872.58018

[2] Chang, K.C., Infinite dimensional Morse theory and multiple solution problems, Progr. nonlinear differential equations appl.,

vol. 6, (1993)

[3] Gromoll, D.; Meyer, W., On differential functions with isolated point, Topology, 8, 361-369, (1969) · Zbl 0212.28903

[4] Li, S.J., Periodic solutions of non-autonomous second order systems with superlinear terms, Differential integral equations, 5, 1419-1424, (1992) · Zbl 0757.34021

[5] Li, S.J.; Liu, J.Q., Some existence theorems on multiple critical points and their applications, Kexue tongbao (Chinese), 17, 1025-1027, (1984)

[6] Li, S.J.; Su, J.B., Existence of multiple critical points for an asymptotically quadratic functional with applications, Abstr. appl. anal., 1, 277-289, (1996) · Zbl 0938.58013

[7] Liu, J.Q., A Morse index for a saddle point, J. systems sci. math. sci., 2, 32-39, (1989) · Zbl 0732.58011

[8] Mawhin, J.; Willem, M., Critical point theory and Hamiltonian systems, (1989), Springer-Verlag New York, Berlin, Heidelberg · Zbl 0676.58017

[9] Rabinowitz, P.H., Some critical point theorems and applications to semilinear elliptic partial differential equations, Ann. sc. norm. super. Pisa cl. sci., 4, 5, 215-223, (1978) · Zbl 0375.35026

[10] Rabinowitz, P.H., Periodic solutions of Hamiltonian systems, Comm. pure appl. math., 31, 157-184, (1978) · Zbl 0358.70014

[11] Rabinowitz, P.H., Minimax methods in critical point theory with applications to differential equations, CBMS reg. conf. ser. math., vol. 65, (1986), AMS Providence, RI · Zbl 0609.58002

[12] Rabinowitz, P.H.; Su, J.B.; Wang, Z.-Q., Multiple solutions of superlinear elliptic equations, Rend. lincei mat. appl., 18, 97-108, (2007) · Zbl 1223.35173

[13] Su, J.B., Multiplicity results for asymptotically linear elliptic problems at resonance, J. math. anal. appl., 278, 397-408, (2003) · Zbl 1290.35109

[14] Su, J.B., Nontrivial critical points for asymptotically quadratic functional at resonance, (), 225-234 · Zbl 1048.58009

[15] Su, J.B.; Zhao, L.G., Multiple periodic solutions of ordinary differential equations with double resonance, Nonlinear anal., 70, 1520-1527, (2009) · Zbl 1219.34056

[16] Wang, Z.-Q., On a superlinear elliptic equation, Ann. inst. H. Poincaré anal. non lineaire, 8, 43-57, (1991) · Zbl 0733.35043