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Periodic solutions of delay impulsive differential equations. (English) Zbl 1242.34134

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The problem studied is the following class of delay differential equations in a Banach space $(X, \|\cdot\|)$,

$$u'(t) + Au(t) = f(t, u(t), u_t), \quad t > 0, \quad t \neq t_k,$$

subject to the initial value $u_0 = \phi$ and the impulse conditions $\Delta u(t_i) = I_i(u(t_i))$, $i = 1, 2, \ldots$, where $0 < t_1 < t_2 < \cdots < \infty$, A is an unbounded operator, $r > 0$, $u_t(s) = u(t + s)$, for $s \in [-r, 0]$ and $\Delta u(t_i)$ denotes the jump of u at the instant t_i.

For a T-periodic function in the first variable f it is proved that, if the solutions to the above-mentioned problems are ultimately bounded, then there exists a T-periodic solution for a certain initial function ϕ. This result is deduced from the Arzelà-Ascoli theorem, which guarantees compactness for a certain operator of interest, and Horn's fixed point theorem, by imposing suitable conditions on the function f, the impulse functions I_i and the impulse instants t_i, and assuming some compactness hypotheses and the existence and uniqueness of mild solutions for each initial value problem on the interval $[0, \infty)$.

The study extends some previous results about non-impulsive equations and similar impulsive ordinary differential equations.

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MSC:

34K30 Functional-differential equations in abstract spaces
34K13 Periodic solutions to functional-differential equations
34K45 Functional-differential equations with impulses
47N20 Applications of operator theory to differential and integral equations

Keywords:

impulsive differential equations; delay differential equations; periodic solutions; fixed points

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