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**Characterization of d.c. Functions in terms of quasidifferentials.** (English) Zbl 1229.90137

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 74, No. 17, 6781-6787 (2011).

Summary: A characterization of d.c. functions $f:\Omega\to\mathbb{R}$ in terms of the quasidifferentials of $f$ is obtained, where $\Omega$ is an open convex set in a real Banach space. Recall that $f$ is called d.c. (difference of convex) if it can be represented as a difference of two finite convex functions. The relation of the obtained results with known characterizations is discussed, specifically the ones from [*R. Ellaia* and *A. Hassouni*, Optimization 22, No.3, 401–416 (1991; Zbl 0734.49005)] in the finite-dimensional case and [*A. Elhilali Alaoui*, Ann. Sci. Math. Qué. 20, No.1, 1–13 (1996; Zbl 0915.49014)] in the case of a Banach space.

## MSC:

90C26 Nonconvex programming, global optimization
26B25 Convexity of real functions of several variables, generalizations
49J52 Nonsmooth analysis




## Keywords:

generalized convexity; d.c. functions; quasidifferentials; characterization of d.c. functions

**Full Text:** DOI


## References:


[1] M.L. Bougeard, Contribution à la la thé orie de Morse en dimension finie, Ph.D. Thesis, Université de Paris IX, 1978.

[2] A. Pommellet, Analyse convexe et théorie de Morse, Ph.D. Thesis, Université de Paris IX, 1982.

[3] Penot, J.P.; Bougeard, M.L., Approximation and decomposition properties of some classes of locally D.C. functions, Math. program. ser. A, 41, 195-227, (1988) · Zbl 0666.49005

[4] Alexandroff, A.D., Almost everywhere existence of the second differential of a convex function and some properties of convex surfaces connected with it, Leningrad state univ. annals [uchenye zapiski] math. ser., 6, 3-35, (1939)

[5] Alexandroff, A.D., Surfaces represented by the difference of convex functions, Dokl. akad. nauk SSSR (NS), 72, 613-616, (1950), (in Russian)

[6] Zallgaller, V.A., On the representation of a function of two variables as the difference of convex functions, Vestn. leningr. univ. ser. mat. mekh., 18, 44-45, (1963), (in Russian)

[7] Landis, E.M., On functions representable as the difference of two convex functions, Dokl. akad. nauk SSSR, 80, 9-11, (1951)

[8] Hartman, P., On functions representable as a difference of convex functions, Pacific J. math., 9, 707-713, (1959) · Zbl 0093.06401

[9] Hiriart-Urruty, J.-B., Generalized differentiability, duality and optimization for problems dealing with differences of convex functions, (), 37-70 · Zbl 0591.90073

[10] Yomdin, Y., On functions representable as a supremum of a family of smooth functions, SIAM J. math. anal., 14, 239-246, (1983) · Zbl 0524.26010

[11] Amahroq, T.; Elhilali Alaoui, A., Mosco-convergence de suites de fonctions DC et conditions d'optimalité du second ordre, Extracta math., 10, 119-129, (1995)

[12] Glover, B.M.; Jeyakumar, V.; Oettli, W., Solvability theorems for classes of difference convex functions, Nonlinear anal., 22, 1191-1200, (1994) · Zbl 0812.49018

[13] Jeyakumar, V.; Glover, B.M., Characterizing global optimality for DC optimization problems under convex inequality constraints, J. global optim., 8, 171-187, (1996) · Zbl 0855.90100

[14] Jeyakumar, V.; Rubinov, A.M.; Glover, B.M.; Ishizuka, Y., Inequality systems and global optimization, J. math. anal. appl., 202, 900-919, (1996) · Zbl 0856.90128

[15] Laghdir, M., Optimality conditions in DC-constrained optimization, Acta math. Vietnam., 30, 169-179, (2005) · Zbl 1155.90474

[16] Shapiro, A., On optimality conditions in quasidifferentiable optimization, SIAM J. control optim., 22, 610-617, (1984) · Zbl 0561.49014

[17] Thach, P.T.; Konno, H., On the degree and separability of nonconvexity and applications to optimization problems, Math. program. ser. A, 77, 23-47, (1997) · Zbl 0891.90137

[18] Tuy, H., Global minimization of a difference of two convex functions, Math. program. stud., 30, 150-182, (1987) · Zbl 0619.90061

[19] Lemaire, B.; Volle, M., Duality in DC programming, (), 331-345 · Zbl 0961.90127

[20] Martínez-Legaz, J.-E.; Volle, M., Duality in D.C. programming: the case of several D.C. constraints, J. math. anal. appl., 237, 657-671, (1999) · Zbl 0946.90064

[21] Martínez-Legaz, J.-E.; Volle, M., Duality for d.c. optimization over compact sets, (), 139-146 · Zbl 1041.90067

[22] Volle, M., Duality principles for optimization problems dealing with the difference of vector-valued convex mappings, J. optim. theory appl., 114, 223-241, (2002) · Zbl 1041.90068

[23] Auchmuty, G., Duality algorithms for nonconvex variational principles, Numer. funct. anal. optim., 10, 211-264, (1989) · Zbl 0646.49023

[24] Moudafi, A., Convergence of a proximal-type method for DC functions, J. appl. funct. anal., 1, 285-291, (2006) · Zbl 1137.49017

[25] Moudafi, A.; Maingé, P.-E., On the convergence of an approximate proximal method for DC functions, J. comput. math., 24, 475-480, (2006) · Zbl 1104.65058

[26] Sun, W.Y.; Sampaio, R.J.B.; Candido, M.A.B., Proximal point algorithm for minimization of DC function, J. comput. math., 21, 451-462, (2003) · Zbl 1107.90427

[27] Azé, D., An example of stability for the minima of a sequence of DC functions: homogenization for a class of nonlinear sturm – liouville problems, Math. program. ser. A, 41, 127-140, (1988) · Zbl 0825.49006

[28] Bui, A.; Bui, M.; Tuy, H., A nonconvex optimization problem arising from distributed computing, Mathematica, 43, 66, 151-165, (2001) · Zbl 1097.90565

[29] Blanquero, R.; Carrizosa, E., Optimization of the norm of a vector-valued DC function and applications, J. optim. theory appl., 107, 245-260, (2000) · Zbl 1064.90034

[30] Melzer, D., On the expressibility of piecewise-linear continuous functions as the difference of two piecewise-linear convex functions, Math. program. stud., 29, 118-134, (1986) · Zbl 0624.49006

[31] Bougeard, M.L., Morse theory for some lower-$C^2$ functions in finite dimension, Math. program. ser. A, 41, 141-159, (1988) · Zbl 0646.49029

[32] Ellaia, R.; Hassouni, A., Characterization of nonsmooth functions through their generalized gradients, Optimization, 22, 401-416, (1991) · Zbl 0734.49005

[33] Elhilali Alaoui, A., Caractérisation des fonctions DC, Ann. sci. math. Québec, 20, 1-13, (1996) · Zbl 0915.49014

[34] Zelený, M., An example of a $C^{1, 1}$ function, which is not a d.c. function, Comment. math. univ. carolin., 43, 149-154, (2002) · Zbl 1090.46012

[35] Duda, J.; Veselý, L.; Zajíček, L., On d.c. functions and mappings, Atti semin. mat. fis. univ. modena, 51, 111-138, (2003) · Zbl 1072.46025

[36] Demyanov, V.F.; Rubinov, A.M., An introduction to quasidifferentiable calculus, (), 1-30 · Zbl 0978.49016

[37] Zălinescu, C., Convex analysis in general vector spaces, (2002), World Scientific Publishing Co., Inc. River Edge, NJ · Zbl 1023.46003

[38] Rockafellar, R.T., ()

[39] Bruckner, A.M.; Bruckner, J.B.; Thompson, B.S., Real analysis, (1997), Prentice Hall Upper Saddle River, New Jersey · Zbl 0872.26001