Roulleau, Xavier

The Fano surface of the Fermat cubic threefold, the del Pezzo surface of degree 5 and a ball quotient. (English) Zbl 1279.14049

Proc. Am. Math. Soc. 139, No. 10, 3405-3412 (2011).

Let S be a smooth projective surface with ample canonical bundle and let D be a reduced simple normal crossing divisor on S (maybe $D = 0$). It is a well known fact that the (logarithmic) Chern numbers $\bar{c}_1^2$ and $\bar{c}_2$ of $S' = S - D$ satisfy

$$\bar{c}_1^2 \leq 3\bar{c}_2\,,$$

and the equality holds if and only if S' is a ball quotient. Few constructions of surfaces with Chern ratio $\frac{\bar{c}_1^2}{\bar{c}_2} = 3$ are known.

In the paper under review, the author considers the Fano surface S (which is a surface of general type) parameterizing lines on the Fermat cubic threefold. He shows that the divisor D consists of 12 disjoint elliptic curves and that S' is a ball quotient with log Chern numbers $\bar{c}_1^2 = 3\bar{c}_2 = 3^4$.

Using a result of [*M. Namba*, Branched coverings and algebraic functions. Pitman Research Notes in Mathematics Series, 161. Harlow: Longman Scientific & Technical; New York: John Wiley & Sons Ltd. (1987; Zbl 0706.14017)] the author proves that there exist a degree 3^4 ramified cover $\eta : S \to \mathcal{H}_1$ branched with order 3 over the ten (-1)-curves of $\mathcal{H}_1$, the del Pezzo surface of degree 5. Moreover, there exists an étale map $\kappa : \mathcal{H}_3 \to S$ that is a quotient of $\mathcal{H}_3$ by an automorphism of order 3, where $\eta_3 : \mathcal{H}_3 \to \mathcal{H}_1$ is a degree 3^5 cover branched over the ten (-1)-curves of $\mathcal{H}_1$ with order 3 constructed by *F. Hirzebruch* [Progr. Math. 36, 113–140 (1983; Zbl 0527.14033)].

In analogy with a result of *T. Yamazaki* and *M. Yoshida* [Math. Ann. 266, 421–431 (1984; Zbl 0513.14008)], the author shows that the surface $\mathcal{T} = \kappa^{-1}S' \subset \mathcal{H}_3$ is a ball quotient: $\mathcal{T} \cong \mathbb{B}_2/\Lambda$, where Λ is the commutator group of the Deligne-Mostow lattice associated to the 5-tuple $(1/3, 1/3, 1/3, 1/3, 2/3)$ [*P. Deligne* and *G. D. Mostow*, Publ. Math., Inst. Hautes Étud. Sci. 63, 5–89 (1986; Zbl 0615.22008)].

Reviewer: Davide Frapporti (Bayreuth)

MSC:

14J29	Surfaces of general type
14J25	Special surfaces
22E40	Discrete subgroups of Lie groups

Keywords:

algebraic surfaces; ball lattices; orbifolds; Fano surfaces of cubic threefolds; degree 5 del Pezzo surface

Full Text: DOI arXiv

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