Kim, Sunghoon; Lee, Ki-Ahm

Viscosity method for homogenization of parabolic nonlinear equations in perforated domains. (English) Zbl 1229.35010

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The main purpose of the paper is to establish homogenization results for parabolic nonlinear equations in perforated domains. The authors start with the obstacle problem $\Delta u_\varepsilon - u_t \le 0$ in $\Omega \times (0,T]$, with $u_\varepsilon \ge \varphi_\varepsilon$ in $\Omega \times (0,T]$, the solution starting from an initial data g at $t = 0$ and satisfying homogeneous Dirichlet boundary conditions on $\partial\Omega \times (0,T]$. Here Ω is a smooth, bounded and connected domain of $\mathbb{R}^n$ and φ_ε is taken as $\varphi\chi_{\mathcal{T}_{a_\varepsilon}}$ where φ is a smooth function which is negative on the boundary $\partial\Omega \times (0,T]$, and $\mathcal{T}_{a_\varepsilon}$ is the union of ε-cells from which have been removed the spherical balls centered at the center of the cells and of radius a_ε.

The first main result of the paper describes the asymptotic behaviour of the least viscosity super-solution of this obstacle problem. The authors distinguish between three cases according to the decay rates of a_ε. The main step of the proof consists of introducing the solution $u_{\varepsilon,\delta}$ of an approximate penalized problem obtained when introducing in the previous equation the penalized term $\beta_\delta(u_{\varepsilon,\delta}(x,t) - \varphi_\varepsilon(x,t))$, where β_δ is some penalty function. The last part of the paper deals with a porous medium $\Delta u_\varepsilon^m - \partial_t u_\varepsilon = 0$ posed in the perforated domain with homogeneous Dirichlet boundary conditions on $\partial\Omega \times (0,T]$. Here $m \in (1,\infty)$ and the solution starts from an initial data $g_\varepsilon = g\xi$ where $g \in C_0^\infty(\Omega)$ and $\xi \in C^\infty$ is an ε-periodic function which is the solution of a Laplace equation in a perforated domain. For this porous medium equation, the authors perform the transformation $v_\varepsilon = u_\varepsilon^m$ and establish the equation satisfied by v_ε. The main result here builds the limit of v_ε when ε goes to 0.

Reviewer: Alain Brillard (Riedisheim)

MSC:

35B27 Homogenization in context of PDEs; PDEs in media with periodic structure
35K55 Nonlinear parabolic equations
35K65 Degenerate parabolic equations
35D40 Viscosity solutions to PDEs
35K85 Unilateral problems for linear parabolic equations and variational inequalities with linear parabolic operators
35K20 Initial-boundary value problems for second-order parabolic equations

Keywords:

obstacle problem; corrector; porous medium equation; approximate penalized problem

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