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Isolated hypersurface singularities and special polynomial realizations of affine quadrics.
(English) [Zbl 1274.32018](#)
J. Geom. Anal. 21, No. 3, 767-782 (2011).

Let \mathcal{O}_n be the ring of germs of holomorphic functions on (\mathbb{C}^n, o) and let $V = \{f = 0\} \subset (\mathbb{C}^n, o)$ with $f \in \mathcal{O}_n$. Then the quotient ring $A(V) := \mathcal{O}_n / (f, \partial f / \partial z_1, \dots, \partial f / \partial z_n)$ is called the moduli algebra of the germ of the hypersurface singularity (V, o) . By the theorem of *J. N. Mather* and *S. S.-T. Yau* [*Invent. Math.* 69, 243–251 (1982; [Zbl 0499.32008](#))], two germs of isolated hypersurface singularities in (\mathbb{C}^n, o) are biholomorphically equivalent if and only if their moduli algebras are isomorphic as \mathbb{C} -algebras. In the paper under review, the authors study the equivalence problem for admissible algebras, i.e., the maximal ideals of Gorenstein Artin algebras, and apply the results to the biholomorphic equivalence problem for quasi-homogeneous hypersurface singularities. In fact, the authors prove that it can be reduced to the linear equivalence problem for certain polynomials arising from the moduli algebras; furthermore these polynomials are completely determined by their quadratic and cubic terms. The last section includes concrete examples in which the equivalence problems in families of hypersurfaces are discussed.

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MSC:

[32S25](#) Complex surface and hypersurface singularities
[13H10](#) Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)
[14B05](#) Singularities in algebraic geometry

Cited in **2** Reviews
Cited in **6** Documents

Keywords:

quasi-homogeneous singularities; isolated hypersurface singularities; moduli algebra; Gorenstein algebras

Full Text: [DOI](#) [arXiv](#)

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