Wu, C. T.; **Park, C. K.**; **Chen, J. S.**

A generalized approximation for the meshfree analysis of solids. (English) Zbl 1217.74150

Int. J. Numer. Methods Eng. 85, No. 6, 693-722 (2011).

Summary: This paper presents a new approach in the construction of meshfree approximations as well as the weak Kronecker-delta property at the boundary, referred to as a generalized meshfree (GMF) approximation. The GMF approximation introduces an enriched basis function in the original Shepard's method. This enriched basis function is introduced to meet the linear or higher order reproducing conditions and at the same time to offer great flexibility on the control of the smoothness and convexity of the approximation. The construction of the GMF approximation can be viewed as a special root-finding scheme of constraint equations that enforces that the basis functions are corrected and the reproducing conditions with certain orders are satisfied within a set of nodes. By choosing different basis functions, various convex and non-convex approximations including moving least-squares (MLS), reproducing kernel (RK), and maximum entropy (ME) approximations can be obtained. Furthermore, the basis function can also be translated or blended with other functions to generate a particular approximation for a special purpose. One application in this paper is to incorporate a blending function at the boundary based on the concept of local convexity for the non-convex approximation, such as MLS, to acquire the weak Kronecker-delta property. To achieve the higher order GMF approximation, two possible methods are also introduced. Several examples are presented to examine the effectiveness of various GMF approximations.

MSC:

74S30 Other numerical methods in solid mechanics (MSC2010)

Keywords:

meshfree method; convex approximation; non-convex approximation; basis function; Kronecker-delta property; root-finding scheme

Software:

Mfree2D

Full Text: DOI

References:

[1] Fasshauer, Handbook of Theoretical and Computational Nanotechnology pp 33– (2006)

[2] Hardy, Multiquadric equations of topography and other irregular surfaces, Journal of Geophysical Research 176 pp 1905– (1971) · doi:10.1029/JB076i008p01905

[3] Duchon, Lecture Notes in Mathematics, in: Constructive Theory of Functions of Several Variables pp 85– (1977) · doi:10.1007/BFb0086566

[4] Powell, Advances in Numerical Analysis pp 303– (1992)

[5] Madych, Bounds on multivariate polynomials and exponential error estimates for multiquadric interpolation, Journal of Approximation Theory 70 pp 94– (1992) · Zbl 0764.41003 · doi:10.1016/0021-9045(92)90058-V

[6] Madych, Multivariate interpolation and conditionally positive definite functions. II, Mathematics of Computation 54 (189) pp 211– (1990) · Zbl 0859.41004 · doi:10.1090/S0025-5718-1990-0993931-7

[7] Hu, Weighted radial basis collocation method for boundary value problems, International Journal for Numerical Methods in Engineering 69 pp 2736– (2007) · Zbl 1194.74525 · doi:10.1002/nme.1877

[8] Chen, Reproducing kernel enhanced local radial basis collocation method, International Journal for Numerical Methods in Engineering 75 pp 600– (2008) · Zbl 1195.74278 · doi:10.1002/nme.2269

[9] Belikov, The non-Sibsonian interpolation: a new method of interpolation of the values of a function on an arbitrary set of points, Computational Mathematics and Mathematical Physics 37 (1) pp 9– (1997) · Zbl 0948.65005

[10] Liu, Mesh Free Methods: Moving Beyond the Finite Element Method (2002) · doi:10.1201/9781420040586

[11] Gu, Moving kriging interpolation and element-free Galerkin method, International Journal for Numerical Methods in Engineering 56 pp 1– (2003) · Zbl 1062.74652 · doi:10.1002/nme.553

[12] Belytschko, Meshless methods: an overview and recent developments, Computer Methods in Applied Mechanics and Engineering 139 pp 3– (1996) · Zbl 0891.73075 · doi:10.1016/S0045-7825(96)01078-X

[13] Belytschko, Element-free Galerkin methods, International Journal for Numerical Methods in Engineering 37 (2) pp 229– (1994) · Zbl 0796.73077 · doi:10.1002/nme.1620370205

[14] Lancaster, Surfaces generated by moving least squares methods, Mathematics of Computation 37 pp 141– (1981) · Zbl 0469.41005 · doi:10.1090/S0025-5718-1981-0616367-1

[15] Chen, Reproducing kernel particle methods for large deformation analysis of non-linear structures, Computer Methods in Applied Mechanics and Engineering 139 pp 195– (1996) · Zbl 0918.73330 · doi:10.1016/S0045-7825(96)01083-3

[16] Liu, Reproducing kernel particle methods, International Journal for Numerical Methods in Fluids 20 pp 1081– (1995) · Zbl 0881.76072 · doi:10.1002/fld.1650200824

[17] Lucy, A numerical approach to testing the fission hypothesis, The Astronomical Journal 82 pp 1013– (1977) · doi:10.1086/112164

[18] Lu, A new implementation of the element free Galerkin method, Computer Methods in Applied Mechanics and Engineering 113 pp 397– (1994) · Zbl 0847.73064 · doi:10.1016/0045-7825(94)90056-6

[19] Gosz, Admissible approximations for essential boundary conditions in the reproducing kernel particle method, Computational Mechanics 19 pp 120– (1996) · Zbl 0889.73078 · doi:10.1007/BF02824850

[20] Chen, New boundary condition treatments in meshfree computation of contact problems, Computer Methods in Applied Mechanics and Engineering 187 pp 441– (2000) · Zbl 0980.74077 · doi:10.1016/S0045-7825(00)80004-3

[21] Kaljevic, An improved element free Galerkin formulation, International Journal for Numerical Methods in Engineering 40 pp 2953– (1997) · Zbl 0895.73079 · doi:10.1002/(SICI)1097-0207(19970830)40:16<2953::AID-NME201>3.0.CO;2-S

[22] Belytschko, A coupled finite element-element-free Galerkin method, Computational Mechanics 17 pp 186– (1995) · Zbl 0840.73058 · doi:10.1007/BF00364080

[23] Huerta, Enrichment and coupling of the finite element and meshless method, International Journal for Numerical Methods in Engineering 48 pp 1615– (2000) · Zbl 0976.74067 · doi:10.1002/1097-0207(20000820)48:11<1615::AID-NME883>3.0.CO;2-S

[24] Jaynes, Information theory and statistical mechanics, Physical Review 106 (4) pp 620– (1957) · Zbl 0084.43701 · doi:10.1103/PhysRev.106.620

[25] Krongauz, Enforcement of essential boundary conditions in meshless approximations using finite elements, Computer Methods in Applied Mechanics and Engineering 131 pp 133– (1996) · Zbl 0881.65098 · doi:10.1016/0045-7825(95)00954-X

[26] Rabczuk, Coupling of mesh-free methods with finite elements: basic concepts and test results, Communications in Numerical Methods in Engineering 22 pp 1031– (2006) · Zbl 1109.65082 · doi:10.1002/cnm.871

[27] Arroyo, Local maximum-entropy approximation schemes: a seamless bridge between finite elements and meshfree methods, International Journal for Numerical Methods in Engineering 65 pp 2167– (2006) · Zbl 1146.74048 · doi:10.1002/nme.1534

[28] Sukumar, Construction of polygonal interpolants: a maximum entropy approach, International Journal for Numerical Methods in Engineering 61 pp 2159– (2004) · Zbl 1073.65505 · doi:10.1002/nme.1193

[29] Sukumar, Overview and construction of meshfree basis functions: from moving least squares to entropy approximants, International Journal for Numerical Methods in Engineering 70 pp 181– (2007) · Zbl 1194.65149 · doi:10.1002/nme.1885

[30] Shannon, A mathematical theory of communication, The Bell Systems Technical Journal 27 pp 379– (1948) · Zbl 1154.94303 · doi:10.1002/j.1538-7305.1948.tb01338.x

[31] Tagliani, Existence and stability of a discrete probability distribution by maximum entropy approach, Applied Mathematics and Computation 110 pp 105– (2000) · Zbl 1017.62011 · doi:10.1016/S0096-3003(98)00034-4

[32] Cyron, Smooth, second order, non-negative meshfree approximants selected by maximum entropy, International Journal for Numerical Methods in Engineering 79 pp 1605– (2009) · Zbl 1176.74208 · doi:10.1002/nme.2597

[33] Hobson, The entropic prior for distributions with positive and negative values, Monthly Notices of the Royal Astronomical Society 298 pp 905– (1998) · doi:10.1046/j.1365-8711.1998.01707.x

[34] Shepard D A two-dimensional interpolation function for irregularly-spaced data 517 524 10.1145/800186.810616

[35] Babuska, The partition of unity method, International Journal for Numerical Methods in Engineering 40 pp 727– (1997) · Zbl 0949.65117 · doi:10.1002/(SICI)1097-0207(19970228)40:4<727::AID-NME86>3.0.CO;2-N

[36] Han, Error analysis of the reproducing kernel particle method, Computer Methods in Applied Mechanics and Engineering 190 pp 6157– (2001) · Zbl 0992.65119 · doi:10.1016/S0045-7825(01)00214-6

[37] Boyd, Convex Optimization (2004) · doi:10.1017/CBO9780511804441

[38] Chen, A stabilized conforming nodal integration for Galerkin mesh-free methods, International Journal for Numerical Methods in Engineering 50 pp 435– (2001) · Zbl 1011.74081 · doi:10.1002/1097-0207(20010120)50:2<435::AID-NME32>3.0.CO;2-A

[39] Timoshenko, Theory of Elasticity (1970)