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Cannon-Thurston maps, i-bounded geometry and a theorem of McMullen. (English)

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Let G be a torsion-free finitely generated Kleinian group, and G_0 a geometrically finite group which has an isomorphism to G preserving the parabolicity and inducing a homeomorphism between the corresponding hyperbolic 3-manifolds. A Cannon-Thurston map is an equivariant continuous map from the limit set of G_0 to that of G. Cannon and Thurston showed the existence of such a map when G is a doubly-degenerate Kleinian surface group corresponding to a fibre of the mapping torus with a pseudo-Anosov monodromy. It was conjectured by Thurston that Cannon-Thurston maps exist for general finitely generated Kleinian groups. Based on Minsky's work, Klarreich showed the existence of Cannon-Thurston maps in the case when G has bounded geometry (i.e. when there is a positive lower bound for the injectivity radii for the corresponding hyperbolic 3-manifold), and McMullen showed the same for the case when G_0 is a once-punctured surface group without assumption of bounded geometry.

Recently the author announced a proof of Thurston's conjecture above for finitely generated Kleinian groups in general. In this expository paper under review, he explains his argument under the assumption that G has "i-bounded geometry". Fixing a positive constant ϵ less than the three-dimensional Margulis constant, a Kleinian group G is said to have i-bounded geometry if there is an upper bound for the geodesic length of meridians on the boundary of every ϵ-Margulis tube in the corresponding hyperbolic 3-manifold $\mathbb{H}^3/G$. This is a weaker condition than having bounded geometry. The author's proof relies on Minsky's bi-Lipschitz model manifolds which were introduced to prove the ending lamination conjecture.

For the entire collection see [Zbl 1213.35007].

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MSC:

57M50 General geometric structures on low-dimensional manifolds
30F30 Differentials on Riemann surfaces
30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)
57M60 Group actions on manifolds and cell complexes in low dimensions

Keywords:

Cannon-Thurston map; Kleinian group; hyperbolic 3-manifold; limit set

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