Clozel, Laurent (ed.); Harris, Michael (ed.); Labesse, Jean-Pierre (ed.); Ngô, Bao-Châu (ed.)

Stabilization of the trace formula, Shimura varieties, and arithmetic applications. Volume 1: On the stabilization of the trace formula. (English) Zbl 1255.11027

Somerville, MA: International Press (ISBN 978-1-57146-227-5/pbk). xiv, 527 p. (2011).

Being the first volume of a series of books on the arithmetic theory of automorphic forms, this book has as subject the stabilization of the trace formula. The greater part of the sixteen chapters is expository, the last three chapters contain new results.

In the first chapter (p. 3) M. Harris explains what the stabilization of the (elliptic part of the) trace formula is and how it can be achieved. Here the essential notions are discussed, results stated without proofs. The second chapter (p. 49), by J.-P. Labesse, is also introductory and is mainly concerned with groups over $\mathbb{R}$ and $\mathbb{C}$, in particular $\mathrm{SL}(2,\mathbb{R})$. After this follow seven chapters on endoscopy and transfer:

A review by D. Renard of Shelstad's results on endoscopy for real reductive groups (p. 95).

P.-H. Chaudouard (p. 145) explains Waldspurger's article where the existence of endoscopic transfer is deduced from the validity of the fundamental lemma for Lie algebras (Chaudouard defines endoscopic data without using the terminology of dual groups).

The next chapter (p. 181), on the twisted fundamental lemma, is an introduction by J.-L. Waldspurger to his work on transfer for twisted endoscopy.

The proof by Ngô Bao Châu of the fundamental lemma for Lie algebras in positive characteristic is sketched by J.-F. Dat and Ngô Dac Tuan (p. 229). The orbital integrals have a geometric interpretation and the proof is by geometric means (perverse sheaves on the Hitchin fibration). The proof of a crucial step (the "support theorem") is sketched by Ngô Bao Châu in the next chapter (p. 253).

B. Lemaire presents Waldspurger's theorem on independence of characteristic: the fundamental lemma for Lie algebras in positive characteristic implies the lemma in characteristic zero (p. 265).

A different proof of independence of characteristic is given by R. Cluckers, T. Hales and F. Loeser using model theory (p. 309).

The third part of the book is a review of local representation theory of unitary groups to be used in the fourth part, which gives some examples of stable transfer. The four chapters of the third part are written by L. Clozel (p. 351), J. Adams (real unitary groups) (p. 369), A. Mínguez (p-adic groups) (p. 389) and J.-P. Labesse (an explicit description of the transfer factors in a particular case) (p. 411).

The examples in the fourth part of the book concern automorphic representations of $\mathrm{GL}(n)$ over a CM field and related unitary groups. This part is divided into three chapters (p. 425).

Let F be a totally real number field and let E be a totally imaginary quadratic extension of F. The subject of the first chapter (IV A) (p. 429), written by J.-P. Labesse, is base change for automorphic representations of a unitary group with respect to E/F that are cohomological at the archimedean places. The results are deduced from the stabilization of a twisted trace formula, proved here under simplifying hypotheses. (Appendice: un erratum by L. Clozel, p. 471)

The remaining two chapters (IV B,C) are written by L. Clozel, M. Harris and J.-P. Labesse. Chapter IV B (p. 475) develops transfer of cohomological tempered automorphic representations from $U(n)\times U(1)$ to $U(n+1)$, under simplifying hypotheses and using IV A. Chapter IV C (p. 497) uses the results of IV A and IV B to construct the Galois representations that appear in the cohomology of certain Shimura varieties of unitary type.

Contents: Michael Harris, General Introduction: The stable trace formula, Shimura varieties, and arithmetic applications. (vii–xiv)

Section II: Endoscopy and transfer (93 ff.):

[R.II.A] David Renard, Endoscopy for real reductive groups (95–142).

Subsection II.B: The fundamental lemma and transfer (143 ff.)

[C.II.B.1] Pierre-Henri Chaudouard, Le transfert lisse des intégrales orbitales, d'après Waldspurger (145-180).

[W.II.B.2] Jean-Loup Waldspurger, A propos du lemme fondamental tordu (181–226).

Subsection II.C: The fundamental lemma and the Hitchin fibration (227 ff.):

[DT.II.C.1] Jean-François Dat and Ngô Dac Thuan, Lemme Fondamental pour les algèbres de Lie, d'après Ngô Bao Châu (229–252),

[N.II.C.2] Ngô Bao Châu, Decomposition theorem and abelian fibration (253–263),

[L.II.D] Bertrand Lemaire, Endoscopie et changement de caractéristique, d'après Waldspurger (265–308),

[CHL.II.E] Raf Cluckers, Thomas Hales and François Loeser, Transfer principle for the fundamental lemma (309–348).

Section III: Brief review of representation theory (349 ff.):

[C.III.A] Laurent Clozel, Identités de caractères en la place archimédienne (351–368),

[A.III.B] Jeffrey Adams, Discrete series and characters of the component group (369–388),

[M.III.C] Alberto Mínguez, Unramified representations of unitary groups (389–422),

[L.III.D] Jean-Pierre Labesse, Les facteurs de transfert pour les groupes unitaires (411–422).

Section IV: Some examples of stable transfer (423 ff.):

Laurent Clozel, Michael Harris and Jean-Pierre Labesse, Introduction to Section IV (425–428),

[L.IV.A] Jean-Pierre Labesse, Changement de base CM et séries discrètes (429–470),

Laurent Clozel, Appendice: Un erratum (471–474),

[CHL.IV.B] Laurent Clozel, Michael Harris and Jean-Pierre Labesse, Endoscopic transfer (475–496),

[CHL.IV.C] Laurent Clozel, Michael Harris and Jean-Pierre Labesse, Construction of automorphic Galois representations. I (497–523).

Reviewer: J. G. M. Mars (Utrecht)

MSC:

11F72	Spectral theory; trace formulas (e.g., that of Selberg)
11F70	Representation-theoretic methods; automorphic representations over local and global fields
22E50	Representations of Lie and linear algebraic groups over local fields
11-06	Proceedings, conferences, collections, etc. pertaining to number theory
22-06	Proceedings, conferences, collections, etc. pertaining to topological groups
00B15	Collections of articles of miscellaneous specific interest

Keywords:

endoscopy; transfer; fundamental lemma; automorphic representations