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On a class of singular Sturm-Liouville periodic boundary value problems. (English)

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Nonlinear Anal., Real World Appl. 12, No. 4, 2378-2384 (2011).

The authors begin with a brief history of the boundary value problem

$$(1 + e\cos(t))x'' - 2e\sin(t)x' + \lambda\sin(x) = 4e\sin(t), \quad t \in [0, 2\pi],$$

$$x(0) = x(2\pi), \qquad x'(0) = x'(2\pi).$$

This boundary value problem models the periodic oscillations of the axis of a satellite in the plane of the elliptic orbit around its center of mass. Here, $0 \leq e < 1$ is the excentricity of the ellipse and $|\lambda| \leq 3$ is a parameter related to the inertia of the satellite. Keeping in mind the case where $e = 1$ in the above problem, the authors consider the solvability of the singular Sturm-Liouville boundary value problem,

$$\big(p(t)x'(t)\big)' = f(t, x), \quad t \in [0, 2T],$$

$$x(0) = x(2T), \quad x'(0) = x'(2T).$$

They give sufficient conditions on both p and f for the existence of a nontrivial odd solution of the singular Sturm-Liouville boundary value problem. They conclude the main section of the paper with an example and a remark about how the conditions on p can be omitted in the case where the Dirichlet boundary value problem is considered. In the final section of the paper, the authors use their main result to establish the existence of a solution of a modified version of the original problem when $e = 1$.

Reviewer: Eric R. Kaufmann (Little Rock)

MSC:

34B16 Singular nonlinear boundary value problems for ordinary differential equations
34B24 Sturm-Liouville theory
34B40 Boundary value problems on infinite intervals for ordinary differential equations

Keywords:

singular Sturm-Liouville problem; periodic solution; solutions on unbounded intervals

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