Ageev, Alexander; **Benchetrit, Yohann**; **Sebő, András**; **Szigeti, Zoltán**

An excluded minor characterization of Seymour graphs. (English) Zbl 1298.05295

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Summary: A graph G is said to be a Seymour graph if for any edge set F there exist $|F|$ pairwise disjoint cuts each containing exactly one element of F, provided for every circuit C of G the necessary condition $|C\cap F| \leq |C\setminus F|$ is satisfied. Seymour graphs behave well with respect to some integer programs including multiflow problems, or more generally odd cut packings, and are closely related to matching theory.

A first coNP characterization of Seymour graphs has been shown by *A. A. Ageev* et al. [J. Graph Theory 24, No. 4, 357–364 (1997; Zbl 0869.05051)], the recognition problem has been solved in a particular case by *A. M. H. Gerards* [J. Comb. Theory, Ser. B 55, No. 1, 73–82 (1992; Zbl 0810.05056)], and the related cut packing problem has been solved in the corresponding special cases. In this article we show a new, minor-producing operation that keeps this property, and prove excluded minor characterizations of Seymour graphs: the operation is the contraction of full stars, or of odd circuits. This sharpens the previous results, providing at the same time a simpler and self-contained algorithmic proof of the existing characterizations as well, still using methods of matching theory and its generalizations.

For the entire collection see [Zbl 1216.90002].

MSC:

05C83 Graph minors
05C75 Structural characterization of families of graphs

Full Text: DOI

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