Wang, Qiang

On generalized Lucas sequences. (English) Zbl 1246.11039

Brualdi, Richard A. (ed.) et al., Combinatorics and graphs. Selected papers based on the presentations at the 20th anniversary conference of IPM on combinatorics, Tehran, Iran, May 15–21, 2009. Dedicated to Reza Khosrovshahi on the occasion of his 70th birthday. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4865-4/pbk). Contemporary Mathematics 531, 127-141 (2010).

For a fixed primitive $(4k+2)$-th root of unity η, the unsigned generalized Lucas sequence of order k is

$$a_n = \sum_{\substack{t=1 \\ t \text{ odd}}}^{2k} (\eta^t + \eta^{-t})^n.$$

When $k = 2$ this gives the classical Lucas sequence. The signed generalized Lucas sequence of order k is the same sum but over even t.

The author gives formulas for the characteristic polynomials $g_k(x)$ and $f_k(x)$, respectively, of these integer sequences. He notes that $f_k(x)g_k(x) = E_{2k}(x)$, the Dickson polynomial of the second kind, and uses this to obtain factorizations of f_k and g_k. He further gets explicit expressions for the remainder of Dickson polynomials of the first kind divided by g_k. This in turn leads to a characterization of permutation binomials of the form $x^r(x^{es}+1)$.

For the entire collection see [Zbl 1202.05003].

Reviewer: Robert Fitzgerald (Carbondale)

MSC:

11B39 Fibonacci and Lucas numbers and polynomials and generalizations
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Lucas sequences; finite fields; Dickson polynomials; permutation polynomials

Software:

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