

López-Rosa, S.; Montero, J.; Sánchez-Moreno, P.; Venegas, J.; Dehesa, J. S.
Position and momentum information-theoretic measures of a D -dimensional particle-in-a-box. (English) [Zbl 1305.81041](#)
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Summary: The main information-theoretic measures of a one-dimensional particle-in-a-box (also known as the infinite potential well or the infinite square well) in both position and momentum spaces, as well as their associated uncertainty relations, are calculated and discussed. The power and entropic moments, the Shannon, Renyi and Tsallis entropies and the Fisher information together with two composite measures (Fisher-Shannon and LMC shape complexities) are considered. Moreover, the associated information-theoretic spreading lengths, which characterize the spread/delocalization of the particle beyond (but complementarily) the standard deviation, and their corresponding uncertainty relations are given and mutually compared. It is found, in particular, that the Fisher length is the proper measure of uncertainty for the infinite well, mainly because it grasps the oscillatory nature of the wavefunctions. Finally, this study is extended to a D -dimensional box.

MSC:

- [81P45](#) Quantum information, communication, networks (quantum-theoretic aspects) Cited in 4 Documents
- [81Q05](#) Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
- [94A17](#) Measures of information, entropy

Keywords:

[infinite well potential](#); [information theory](#); [entropic moments](#); [Renyi entropy](#); [Tsallis entropy](#); [Shannon entropy](#); [Fisher information](#); [LMC shape complexity](#); [Rydberg states](#); [information-theoretic lengths](#)

Full Text: [DOI](#)

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