Zhang, Zhimin; Yang, Hailiang; Yang, Hu
**On the absolute ruin in a map risk model with debit interest.** (English) Zbl 1229.91171
Adv. Appl. Probab. 43, No. 1, 77-96 (2011).

A Markov-additive risk model $\{U_t, J_t\}$ is considered, where $U_t$ is the surplus and $J_t$ is the state of the Markov process. If the surplus becomes negative, interest at rate $r$ has to be paid for the deficit. The time of absolute ruin $T$ is the first time where the payments for interest are larger than the premium income. The quantity of interest is the discounted penalty function

$$\Phi_{ij}(u) = E_{(u,i)}[e^{-\delta T} w(U_{T-} - c/r, c/r - U_T) I_{J_T = j}] \,,$$

where $c$ is the premium rate, $w$ is a bounded measurable function, and $I$ is the indicator function. The usual integro-differential equations are proved. In a complicated way, the boundary condition $\Phi_{ij}(-c/r)$ is found. It would have been simpler to observe that $\Phi$ is continuous in $-c/r$ and that starting in $u = -c/r$ means that absolute ruin occurs at the first claim time with $U_{T-} = -c/r$.

Heavy-tailed claim sizes are then considered. Four classes are introduced: Subexponential and long-tailed distributions as well subexponential and long-tailed density functions. The asymptotic behaviour of $\Phi$ for the case of subexponential distributions and subexponential densities is calculated.

Unfortunately, the paper contains some errors. For example, in the proof of Lemma 4 it is claimed that $F$ long-tailed implies that also the density $f$ of $F$ is long-tailed. The following simple example gives a long-tailed distribution with an infinitely often differentiable density. Let $h(x) = C\exp\{-1/(1-x^2)\} I_{|x|<1}$ with $C$ chosen such that $\int_{-1}^1 h(x)\mathrm{d}x = 1$. Then

$$f(x) = \frac{1}{2}e^{-x} + \sum_{n=1}^{\infty} nh(2n^2(n+1)(x-n^2)) \,.$$

Since $e^x$ is not heavy-tailed, it does not matter asymptotically. The weight close to $n^2$ is approximately $\frac{1}{2n(n+1)}$. Thus the tail of $F$ is

$$\sum_{n=x}^{\infty} \frac{1}{2n(n+1)} \sim \int_x^{\infty} \frac{1}{2y(y+1)}\mathrm{d}y = \frac{1}{2}\log\frac{x+1}{x} \sim \frac{1}{2x} \,.$$

This proves that $F$ is long-tailed. But for all $y \neq 0$

$$\limsup_{x\to\infty} \frac{f(x+y)}{f(x)} = \limsup_{x\to\infty} \frac{f(x)}{f(x+y)} = \limsup_{x\to\infty} f(x) = \infty \,.$$

Reviewer: Hanspeter Schmidli (Köln)

**MSC:**

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|---|---|
| 91B30 | Risk theory, insurance (MSC2010) |
| 60J28 | Applications of continuous-time Markov processes on discrete state spaces |
| 91B70 | Stochastic models in economics |



**Keywords:**

Markov additive processes; absolute ruin; discounted penalty function; matrix renewal equation; asymptotics; heavy-tailed distribution

**Full Text:** DOI

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