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Finitely additive probability measures on classical propositional formulas definable by Gödel's t-norm and product t-norm. (English) Zbl 1233.03032

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Summary: Suppose that e is any $[0,1]$-valued evaluation of the set of propositional letters. Then, e can be uniquely extended to finitely additive probability product and Gödel measures on the set of classical propositional formulas. Those measures satisfy that the measure of any conjunction of distinct propositional letters is equal to the product or to the minimum of the measures of the propositional letters, respectively. Product measures correspond to the one extreme – stochastic or probabilistic independence of elementary events (propositional letters), while Gödel measures correspond to the other extreme – logical dependence of elementary events. Any linear convex combination of a product measure and a Gödel measure is also a finitely additive probability measure. In that way infinitely many intermediate measures that correspond to various degrees of dependence of propositional letters can be generated. Such measures give certain truth-functional flavor to probability, enabling applications to preferential problems, in particular classifications according to predefined criteria. Some examples are provided to illustrate this possibility. We present the proof-theoretical and the model-theoretical approaches to a probabilistic logic which allows for reasoning about the mentioned types of probabilistic functions. The logical language enables formalization of classification problems with the corresponding criteria expressible as propositional formulas. However, more complex criteria, for example involving arithmetical functions, cannot be represented in that framework. We analyze the well-known problem proposed by Grabisch to illustrate interpretation of such classification problems in fuzzy logic.

MSC:

03B52 Fuzzy logic; logic of vagueness
03B48 Probability and inductive logic

Cited in **3** Documents

Keywords:

probabilistic logic; fuzzy logic; classification problem; product t-norm; Gödel's t-norm

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