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Large time behavior of isentropic compressible Navier-Stokes system in $\mathbb{R}^3$. (English)

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Summary: We consider the long-time behavior and optimal decay rates of global strong solution to three-dimensional isentropic compressible Navier-Stokes (CNS) system in the present paper. When the regular initial data also belong to some Sobolev space $H^l(\mathbb{R}^3) \cap \dot{B}^{-s}_{1,\infty}(\mathbb{R}^3)$ with $l \geq 4$ and $s \in [0, 1]$, we show that the global solution to the CNS system converges to the equilibrium state at a faster decay rate in time. In particular, the density and momentum converge to the equilibrium state at the rates $(1 + t)^{-3/4-s/2}$ in the L^2-norm or $(1 + t)^{-3/2-s/2}$ in the L^∞-norm, respectively, which are shown to be optimal for the CNS system.

MSC:

35Q30 Navier-Stokes equations
76N10 Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics
35B40 Asymptotic behavior of solutions to PDEs
35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs

Keywords:

compressible Navier-Stokes system; optimal decay rate

Full Text: DOI

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