

Graff, Grzegorz; Nowak-Przygodzki, Piotr; Ruiz del Portal, Francisco R.

Local fixed point indices of iterations of planar maps. (English) Zbl 1230.37029

J. Dyn. Differ. Equations 23, No. 1, 213–223 (2011).

Let $f : U \rightarrow \mathbb{R}^2$ be a continuous map on an open subset U of \mathbb{R}^2 , and let p be a fixed point of f which is neither a source nor a sink. Consider $S_p := (\text{ind}(f^n, p))_{n=1}^{\infty}$, the sequence of fixed point indices of the iterates of f . If $\{p\}$ is an isolated invariant set for f then it is proved that S_p is periodic, bounded above by 1, and has infinitely many non-positive terms.

This generalizes a result by *P. Le Calvez* and *J.-C. Yoccoz* [*Ann. Math.* (2) 146, No. 2, 241–293 (1997; [Zbl 0895.58032](#))] which gives a rather precise description of S_p if f is an orientation preserving local homeomorphism. The hypothesis that $\{p\}$ is an isolated invariant set amounts to assuming that there is a compact neighborhood N of p such that any two-sided sequence $(\sigma(n))_{n=-\infty}^{\infty}$ in N satisfying $\sigma(n+1) = f(\sigma(n))$ for all $n \in \mathbb{Z}$ is the constant sequence with value p . Applications are given that concern the existence of minimal maps on the 2-sphere.

The methods employed involve replacing f by a self-map on a suitable space allowing to compute the index at p by the Lefschetz-Hopf theorem. Another ingredient of the proof is a result of *B. Jiang* from Nielsen theory of surfaces.

Reviewer: [Dieter Erle \(Dortmund\)](#)

MSC:

37C25 Fixed points and periodic points of dynamical systems; fixed-point index theory, local dynamics

Cited in 2 Documents

37E30 Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces

37B30 Index theory for dynamical systems, Morse-Conley indices

Keywords:

fixed point index; planar map; periodic point; iterate of a map; Conley index; Nielsen theory; isolated invariant set; minimal map; Lefschetz-Hopf theorem

Full Text: [DOI](#)

References:

- [1] Babenko I.K., Bogatyı S.A.: The behavior of the index of periodic points under iterations of a mapping. *Math. USSR Izv.* 38, 1–26 (1992) · [Zbl 0742.58027](#) · [doi:10.1070/IM1992v038n01ABEH002185](#)
- [2] Bernardes N.: On the set of points with a dense orbit. *Proc. Amer. Math. Soc.* 128(11), 3421–3423 (2000) · [Zbl 0947.37005](#) · [doi:10.1090/S0002-9939-00-05438-1](#)
- [3] Bogatyı, S.A.: Local indices of iterations of a holomorphic mapping. (Russian) *General topology. Spaces and mappings*, 48–61, *Moskov. Gos. Univ., Moscow*, (1989)
- [4] Chow, S.N., Mallet-Parret, J., Yorke, J.A.: A periodic point index which is a bifurcation invariant, *Geometric dynamics (Rio de Janeiro, 1981)*, 109–131, *Springer Lecture Notes in Math.* 1007, Berlin (1983)
- [5] Dold A.: Fixed point indices of iterated maps. *Invent. Math.* 74, 419–435 (1983) · [Zbl 0583.55001](#) · [doi:10.1007/BF01394243](#)
- [6] Fagella N., Llibre J.: Periodic points of holomorphic maps via Lefschetz numbers. *Trans. Amer. Math. Soc.* 352(10), 4711–4730 (2000) · [Zbl 0947.55001](#) · [doi:10.1090/S0002-9947-00-02608-8](#)
- [7] Franks J.: The Conley index and non-existence of minimal homeomorphisms. *Illinois J. Math.* 43(3), 457–464 (1999) · [Zbl 0948.37005](#)
- [8] Franks J., Richeson D.: Shift equivalence and the Conley index. *Trans. Amer. Math. Soc.* 352(7), 3305–3322 (2000) · [Zbl 0956.37010](#) · [doi:10.1090/S0002-9947-00-02488-0](#)
- [9] Graff G., Nowak-Przygodzki P.: Fixed point indices of iterations of C^1 maps in \mathbb{R}^3 . *Discrete Cont. Dyn. Syst.* 16(4), 843–856 (2006) · [Zbl 1185.37043](#) · [doi:10.3934/dcds.2006.16.843](#)
- [10] Graff G., Nowak-Przygodzki P.: Sequences of fixed point indices of iterations in dimension 2. *Univ. Iagel. Acta Math.* XLI,

135–140 (2003) · [Zbl 1061.37016](#)

- [11] Jezierski, J., Marzantowicz, W.: Homotopy methods in topological fixed and periodic points theory, topological fixed point theory and its applications, 3. Springer, Dordrecht (2005) · [Zbl 1085.55001](#)
- [12] Jiang B.J.: Bounds for fixed points on surfaces. *Math. Ann.* 311(3), 467–479 (1998) · [Zbl 0903.55003](#) · [doi:10.1007/s002080050195](#)
- [13] Jiang, B.J.: Lectures on the Nielsen fixed point theory, *Contemp. Math.* 14, Amer. Math. Soc., Providence (1983) · [Zbl 0512.55003](#)
- [14] Le Calvez P.: Dynamique des homomorphismes du plan au voisinage d'un point fixe. *Ann. Sci. École Norm. Sup.* (4) 36(1), 139–171 (2003) · [Zbl 1017.37017](#)
- [15] Le Calvez P., Salazar J.M., Ruiz del Portal F.R.: Fixed point indices of the iterates of \mathbb{R}^3 -homeomorphisms at fixed points which are isolated invariant sets. *J. London Math. Soc.* 82(2), 683–696 (2010) · [Zbl 1209.37020](#) · [doi:10.1112/jlms/jdq050](#)
- [16] Le Calvez P., Yoccoz J.-C.: Un théorème d'indice pour les homéomorphismes du plan au voisinage d'un point fixe. *Annals of Math.* 146, 241–293 (1997) · [Zbl 0895.58032](#) · [doi:10.2307/2952463](#)
- [17] Mischaikow K., Mrozek M.: Conley index. *Handbook of dynamical systems*, vol. 2, pp. 393–460. North-Holland, Amsterdam (2002) · [Zbl 1035.37010](#)
- [18] Marzantowicz W., Przygodzki P.: Finding periodic points of a map by use of a k-adic expansion. *Discrete Contin. Dyn. Syst.* 5, 495–514 (1999) · [Zbl 0965.37015](#) · [doi:10.3934/dcds.1999.5.495](#)
- [19] Mauldin, R.D. (red): *The Scottish book*, Birkhäuser, Boston (1981) · [Zbl 0485.01013](#)
- [20] Ruiz del Portal, F.R., Salazar, J.M.: A Poincaré formula for the fixed point indices of the iterates of arbitrary planar homeomorphisms, *Fixed Point Theory Appl.* Art. ID 323069, pp. 31 (2010) · [Zbl 1196.54068](#)
- [21] Ruiz del Portal F.R., Salazar J.M.: A stable/unstable manifold theorem for local homeomorphisms of the plane. *Ergodic Theory Dynam. Systems* 25(1), 301–317 (2005) · [Zbl 1073.37047](#) · [doi:10.1017/S0143385704000367](#)
- [22] Ruiz del Portal F.R., Salazar J.M.: Fixed point index of iterations of local homeomorphisms of the plane: a Conley index approach. *Topology* 41(6), 1199–1212 (2002) · [Zbl 1009.54043](#) · [doi:10.1016/S0040-9383\(01\)00035-0](#)
- [23] Ruiz del Portal F.R., Salazar J.M.: Indices of the iterates of \mathbb{R}^3 -homeomorphisms at Lyapunov stable fixed points. *J. Diff. Eq.* 244(5), 1141–1156 (2008) · [Zbl 1132.37012](#) · [doi:10.1016/j.jde.2007.08.006](#)
- [24] Shub M., Sullivan P.: A remark on the Lefschetz fixed point formula for differentiable maps. *Topology* 13, 189–191 (1974) · [Zbl 0291.58014](#) · [doi:10.1016/0040-9383\(74\)90009-3](#)
- [25] Zhang G.Y.: Fixed point indices and invariant periodic sets of holomorphic systems. *Proc. Amer. Math. Soc.* 135(3), 767–776 (2007) (electronic) · [Zbl 1121.32007](#) · [doi:10.1090/S0002-9939-06-08821-6](#)
- [26] Zhang G.Y.: Fixed point indices and periodic points of holomorphic mappings. *Math. Ann.* 337(2), 401–433 (2007) · [Zbl 1110.32007](#) · [doi:10.1007/s00208-006-0042-6](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.