

**Leininger, Christopher; Long, Darren D.; Reid, Alan W.**

**Commensurators of finitely generated nonfree Kleinian groups.** (English) Zbl 1237.20044  
Algebr. Geom. Topol. 11, No. 1, 605-624 (2011).

This article was inspired by the question of whether there is a broad generalization of the theorem of Margulis by replacing the finite covolume hypothesis by the weaker assumption that the group only be Zariski dense. Let  $\Gamma$  be a finitely generated torsion-free Kleinian group of the first kind which is not a lattice and  $C(\Gamma)$  be the commensurator of  $\Gamma$ . The authors show that if  $\Gamma$  is not free and contains no parabolic elements then  $C(\Gamma)$  is discrete, furthermore,  $[C(\Gamma) : \Gamma] = \infty$  if and only if  $\Gamma$  is a fiber group, in this case,  $C(\Gamma)$  is a lattice.

Reviewer: [Osman Bizim \(Bursa\)](#)

**MSC:**

- [20H10](#) Fuchsian groups and their generalizations (group-theoretic aspects)
- [30F40](#) Kleinian groups (aspects of compact Riemann surfaces and uniformization)
- [22E40](#) Discrete subgroups of Lie groups
- [57M50](#) General geometric structures on low-dimensional manifolds

Cited in **5** Documents

**Keywords:**

[Kleinian groups](#); [commensurators](#); [finitely generated torsion-free groups](#)

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