Guillemin, Fabrice; **van Leeuwaarden, Johan S. H.**

Rare event asymptotics for a random walk in the quarter plane. (English) Zbl 1210.60100

Queueing Syst. 67, No. 1, 1-32 (2011).

Random walks in a quarter plane have been studied by several authors [see e.g. *G. Fayolle* and *R. Iasnogorodski*, Z. Wahrscheinlichkeitstheor. Verw. Geb. 47, 325–351 (1979; Zbl 0395.68032); *G. Fayolle, R. Iasnogorodski* and *V. Malyshev*, Random walks in the quarter-plane. Algebraic methods, boundary value problems and applications. Berlin: Springer (1999; Zbl 0932.60002); *J. W. Cohen* and *O. J. Boxma*, Boundary value problems in queueing system analysis. Amsterdam - New York - Oxford: North-Holland Publishing Company (1983; Zbl 0515.60092)].

This paper presents new analytic techniques for deriving asymptotic expressions for the occurrence of rare events for a random walk in the quarter plane. The results are applied for tandem queues with Poisson arrivals, exponential service times and coupled processors. The authors derive the functional equation for the bivariate generating function of the queue-lengths and investigate it. For the asymptotic analysis of large queue-lengths they combine the kernel method for functional equations with boundary value problems and singularity analysis.

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MSC:

60K25 Queueing theory (aspects of probability theory)
60F10 Large deviations

Keywords:

boundary value problems; random walks in the quarter plane; rare events; queuing theory; singularity analysis; tail decay rate; large deviations

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