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**Self-similar random fields and rescaled random balls models.** (English) Zbl 1213.60096  
J. Theor. Probab. 23, No. 4, 1110-1141 (2010).

Author's abstract: We study generalized random fields which arise as rescaling limits of spatial configurations of uniformly scattered random balls as the mean radius of the balls tends to 0 or infinity. Assuming that the radius distribution has a power-law behavior, we prove that the centered and renormalized random balls field admits a limit with self-similarity properties. Our main result states that all self-similar, translation- and rotation-invariant Gaussian fields can be obtained through a unified zooming procedure starting from a random balls model. This approach has to be understood as a microscopic description of macroscopic properties. Under specific assumptions, we also get a Poisson-type asymptotic field. In addition to investigating stationarity and self-similarity properties, we give  $L^2$ -representations of the asymptotic generalized random fields viewed as continuous random linear functionals.

Reviewer: [Wolfgang Freudenberg \(Cottbus\)](#)

**MSC:**

[60G60](#) Random fields  
[60G55](#) Point processes (e.g., Poisson, Cox, Hawkes processes)  
[60G18](#) Self-similar stochastic processes  
[60D05](#) Geometric probability and stochastic geometry  
[60G20](#) Generalized stochastic processes  
[60F05](#) Central limit and other weak theorems

Cited in **22** Documents

**Keywords:**

[self-similarity](#); [generalized random field](#); [Poisson point process](#); [fractional Poisson field](#); [fractional Brownian field](#)

**Full Text:** [DOI](#)

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