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Dynamical complexities in the Leslie-Gower predator-prey model as consequences of the Allee effect on prey. (English) Zbl 1202.34079

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Summary: This work deals with the analysis of a predator-prey model derived from the Leslie-Gower type model, where the most common mathematical form to express the Allee effect in the prey growth function is considered.

It is well-known that the Leslie-Gower model has a unique globally asymptotically stable equilibrium point. However, it is shown here the Allee effect significantly modifies the original system dynamics, as the studied model involves many non-topological equivalent behaviors.

None, one or two equilibrium points can exist at the interior of the first quadrant of the modified Leslie-Gower model with strong Allee effect on prey. However, a collapse may be seen when two positive equilibrium points exist.

Moreover, we proved the existence of parameter subsets for which the system can have: a cusp point (Bogdanov-Takens bifurcation), homoclinic curves (homoclinic bifurcation), Hopf bifurcation and the existence of two limit cycles, the innermost stable and the outermost unstable, in inverse stability as they usually appear in the Gause-type predator-prey models.

In contrast, the system modelling an special of weak Allee effect, may include none or just one positive equilibrium point and no homoclinic curve; the latter implies a significant difference between the mathematical properties of these forms of the phenomenon, although both systems show some rich and interesting dynamics.

MSC:

34C23 Bifurcation theory for ordinary differential equations
92D25 Population dynamics (general)

Keywords:

Allee effect; Leslie-Gower predator-prey models; functional response; limit cycle; bifurcations; separatrix curves

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