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On a problem of magnetohydrodynamics in a multi-connected domain. (English)

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Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 74, No. 2, 462-478 (2011).

Summary: We consider the following problem in the MHD approximation: the vessel $\Omega_1 \subset \Omega$ is filled with an incompressible, electrically conducting fluid, and is surrounded by a dielectric or by vacuum, occupying the bounded domain $\Omega_2 = \Omega \setminus \Omega_1$. In Ω we have a magnetic and electric field and the external surface $S = \partial\Omega$ is an ideal conductor. The emphasis in the paper is on when Ω is not simply connected, in which case the MHD system is degenerate. We use Hodge-type decomposition theorems to obtain strong solutions locally in time or global for small enough initial data, and a linearization principle for the stability of a stationary solution.

MSC:

35Q35	PDEs in connection with fluid mechanics
35B35	Stability in context of PDEs
76D03	Existence, uniqueness, and regularity theory for incompressible viscous fluids
76W05	Magnetohydrodynamics and electrohydrodynamics

Keywords:

magnetohydrodynamics; Navier-Stokes equations; stability; Hodge decomposition; Maxwell equations

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