Bürger, Raimund; Karlsen, Kenneth H.; Torres, Héctor; Towers, John D.

Second-order schemes for conservation laws with discontinuous flux modelling clarifier-thickener units. (English) Zbl 1204.65101

Numer. Math. 116, No. 4, 579-617 (2010).

The authors consider mainly a staggered first-order difference scheme and its second-order upgrading for a hyperbolic equation in one spatial variable modelling sediment suspension behaviour. The model contains a discontinuous flux, and in former works of the same authors [Comput. Vis. Sci. 6, No. 2–3, 83–91 (2004; Zbl 1299.76283)] the convergence of the first-order scheme to an entropy solution has been considered. Here they upgrade the scheme to (formal) second order by using corresponding correction terms and two kinds of limiters, a simple minmod and a new one (leading to a flux-total variation diminishing scheme), including here a correction for steady sonic rarefactions, too. For the nonlocal limiter proposed, an algorithm is formulated, its properties investigated and its numerical behaviour illustrated by a number of experiments and compared with the simpler scheme. Finally, an extended model including degenerate diffusion is considered also and solved by operator splitting (combining the former hyperbolic scheme and Crank-Nicolson), exhibiting good numerical results for the discontinuous solutions.

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MSC:

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

conservation law; discontinuous flux; first-order difference scheme; second-order correction; limiters; numerical experiments; sediment suspension; convergence; total variation diminishing scheme; algorithm; operator splitting; Crank-Nicolson; discontinuous solutions

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