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Numerical solution of nonlinear Schrödinger equation by using time-space pseudo-spectral method. (English) Zbl 1195.65137

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Summary: A time-space pseudo-spectral method is proposed for the numerical solution of nonlinear Schrödinger equation. The employed method is based on Chebyshev-Gauss-Lobbato quadrature points. Using the pseudo-spectral differentiation matrices the problem is reduced to a system of nonlinear algebraic equations. However, this method is basically a spectral method, but a subdomain-in-time algorithm is used which yields a smaller nonlinear system to study long-time numerical behavior. Because the time-space pseudo-spectral method has spectral accuracy, we present numerical experiments which show high accuracy of this method for the variant nonlinear Schrödinger equations and also particular attention is paid to the conserved quantities as an indicator of the accuracy.

MSC:

65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

35Q55 NLS equations (nonlinear Schrödinger equations)

Keywords:

nonlinear Schrödinger equation; pseudo-spectral method; soliton; time-space pseudo-spectral method; Chebyshev-Gauss-Lobbato quadrature points; Fourier pseudo-spectral time splitting method; Gross-Pitaveskii equation; numerical experiments

Software:

Matlab

Full Text: DOI

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