Bürger, Raimund; **Karlsen, Kenneth H.**; **Towers, John D.**

An Engquist-Osher-type scheme for conservation laws with discontinuous flux adapted to flux connections. (English) Zbl 1201.35022

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The authors consider the numerical approximation of solutions to the initial value problem

$$\begin{aligned} u_t + \mathcal{F}(x,u)_x &= 0 \quad \text{for } (x,t) \in \mathbb{R} \times (0,T), \\ u(x,0) &= u_0(x) \quad \text{for } x \in \mathbb{R}, \end{aligned}$$

$$\mathcal{F}(x,u) := H(x)f(u) + \big(1 - H(x)\big)g(u) = \begin{cases} f(u) & \text{for } x \geq 0, \\ g(u) & \text{for } x < 0. \end{cases}$$

The main contribution of this paper is a scalar monotone difference scheme, for which the authors prove convergence to an entropy solution of type (A,B). The scheme is simple in the sense that no 2×2 Riemann solver is required. It takes the form of an explicit conservative marching formula on a rectangular grid, where the numerical flux for all cells is the Engquist-Osher (EO) flux, with the exception of the cell interface that is associated with the flux discontinuity, and for which a specific interface flux is used. The interface flux, which is based on a novel modification of the EO flux, is designed to preserve certain steady-state solutions. Some numerical examples are presented.

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MSC:

35A35	Theoretical approximation in context of PDEs
35L65	Hyperbolic conservation laws
65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
35L45	Initial value problems for first-order hyperbolic systems

Keywords:

flux connection; adapted entropy; entropy solution of type (A,B); Heaviside function; scalar monotone difference scheme

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