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Optimal decay rate of the compressible Navier-Stokes-Poisson system in $\mathbb{R}^3$. (English)

Zbl 1205.35201
Arch. Ration. Mech. Anal. 196, No. 2, 681-713 (2010).

The authors study the compressible Navier-Stokes-Poisson system in the whole three-dimensional space which describes, e.g., the transport phenomena of charged particles under the electric field of an electrostatic potential force. They consider classical solutions to the system which exist, e.g., if the initial condition is sufficiently close to a constant equilibrium solution. The aim of the paper is to establish the decay rate in time of certain space norms of the solution. The L^2 norm of the difference of the density to the given constant decays as $t^{-3/4}$ the momentum as $t^{-1/4}$ and the gradient of the electrostatic potential as $t^{-1/4}$, for $t \to +\infty$. It is shown that the decay rate is optimal in view of the fact that also the lower bounds are proved. Finally, also the decay in some L^p norms for $2 \leq p \leq \infty$ is studied. The proofs are based on a careful study of the linearized problem in the Fourier space and on estimates of the corresponding semigroups.

Reviewer: Milan Pokorný (Praha)

MSC:

35Q30 Navier-Stokes equations
76N10 Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics
78A30 Electro- and magnetostatics
76W05 Magnetohydrodynamics and electrohydrodynamics

Keywords:

Navier-Stokes-Poisson system; Cauchy problem; strong solution; time decay

Full Text: DOI arXiv

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