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Uniqueness and non-uniqueness in percolation theory. (English) Zbl 1189.60175
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Summary: This paper is an up-to-date introduction to the problem of uniqueness versus non-uniqueness of infinite clusters for percolation on \mathbb{Z}^d and, more generally, on transitive graphs. For iid percolation on \mathbb{Z}^d , uniqueness of the infinite cluster is a classical result, while on certain other transitive graphs uniqueness may fail. Key properties of the graphs in this context turn out to be amenability and nonamenability. The same problem is considered for certain dependent percolation models – most prominently the Fortuin-Kasteleyn random-cluster model – and in situations where the standard connectivity notion is replaced by entanglement or rigidity. So-called simultaneous uniqueness in couplings of percolation processes is also considered. Some of the main results are proved in detail, while for others the proofs are merely sketched, and for yet others they are omitted. Several open problems are discussed.

MSC:

- 60K35** Interacting random processes; statistical mechanics type models; percolation theory
- 82B43** Percolation
- 60-02** Research exposition (monographs, survey articles) pertaining to probability theory
- 82-02** Research exposition (monographs, survey articles) pertaining to statistical mechanics

Cited in **27** Documents

Keywords:

percolation; uniqueness of the infinite cluster; transitive graphs; amenability

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