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Application of variational iteration method to fractional hyperbolic partial differential equations. (English) Zbl 1190.65185

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Summary: The solution of the fractional hyperbolic partial differential equation is obtained by means of the variational iteration method. Our numerical results are compared with those obtained by the modified Gauss elimination method. Our results reveal that the technique introduced here is very effective, convenient, and quite accurate to one-dimensional fractional hyperbolic partial differential equations. Application of variational iteration technique to this problem has shown the rapid convergence of the sequence constructed by this method to the exact solution.

MSC:

65N99 Numerical methods for partial differential equations, boundary value problems
35L99 Hyperbolic equations and hyperbolic systems
26A33 Fractional derivatives and integrals

Full Text: DOI EuDML

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