Ringler, T. D.; **Thuburn, J.**; **Klemp, J. B.**; **Skamarock, W. C.**

A unified approach to energy conservation and potential vorticity dynamics for arbitrarily-structured C-grids. (English) Zbl 1307.76054

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Summary: A numerical scheme applicable to arbitrarily-structured C-grids is presented for the nonlinear shallow-water equations. By discretizing the vector-invariant form of the momentum equation, the relationship between the nonlinear Coriolis force and the potential vorticity flux can be used to guarantee that mass, velocity and potential vorticity evolve in a consistent and compatible manner. Underpinning the consistency and compatibility of the discrete system is the construction of an auxiliary thickness equation that is staggered from the primary thickness equation and collocated with the vorticity field. The numerical scheme also exhibits conservation of total energy to within time-truncation error. Simulations of the standard shallow-water test cases confirm the analysis and show convergence rates between 1st- and 2nd-order accuracy when discretizing the system with quasi-uniform spherical Voronoi diagrams. The numerical method is applicable to a wide class of meshes, including latitude-longitude grids, Voronoi diagrams, Delaunay triangulations and conformally-mapped cubed-sphere meshes.

Reviewer: Reviewer (Berlin)

MSC:

76M12	Finite volume methods applied to problems in fluid mechanics
76B15	Water waves, gravity waves; dispersion and scattering, nonlinear interaction
65M08	Finite volume methods for initial value and initial-boundary value problems involving PDEs
65M50	Mesh generation, refinement, and adaptive methods for the numerical solution of initial value and initial-boundary value problems involving PDEs

Keywords:

C-grid; Voronoi diagram; potential vorticity; shallow-water equations

Software:

chammp

Full Text: DOI

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