Huang, Z. L.; **Jin, X. L.**; **Lim, C. W.**; **Wang, Y.**

Statistical analysis for stochastic systems including fractional derivatives. (English)

Zbl 1183.70062
Nonlinear Dyn. 59, No. 1-2, 339-349 (2010).

Summary: An analytical scheme to determine the statistical behavior of a stochastic system including two terms of fractional derivative with real, arbitrary, fractional orders is proposed. In this approach, Green's functions obtained are based on a Laplace transform approach and the weighted generalized Mittag-Leffler function. The responses of the system can be subsequently described as a Duhamel integral-type close-form expression. These expressions are applied to obtain the statistical behavior of a dynamical system excited by stationary stochastic processes. The numerical simulation based on the modified Euler method and Monte Carlo approach is developed. Three examples of single-degree-of-freedom system with fractional derivative damping under Gaussian white noise excitation are presented to illustrate application of the proposed method.

MSC:

70L05 Random vibrations in mechanics of particles and systems
26A33 Fractional derivatives and integrals

Keywords:

statistical behavior; fractional derivatives; Laplace transform; Duhamel integral; numerical simulation

Full Text: DOI

References:

[1] Podlubny, I.: Fractional Differential Equations. Academic Press, San Diego (1999) · Zbl 0924.34008

[2] West, B.J., Bologna, M., Grigolini, P.: Physics of Fractal Operators. Springer, New York (2003)

[3] Lim, S.C.: Fractional derivative quantum fields at positive temperature. Physica A 363, 269–281 (2006) · doi:10.1016/j.physa.2005.08.005

[4] Bagley, R.L., Torvik, P.J.: A theoretical basis for the application of fractional calculus to viscoelasticity. J. Rheol. 27, 201–210 (1983) · Zbl 0515.76012 · doi:10.1122/1.549724

[5] Bagley, R.L., Torvik, P.J.: Fractional calculus: a different approach to the analysis of viscoelastically damped structures. AIAA J. 21, 741–748 (1983) · Zbl 0514.73048 · doi:10.2514/3.8142

[6] Bagley, R.L., Torvik, P.J.: Fractional calculus in the transient analysis of viscoelastically damped structures. AIAA J. 23, 918–925 (1985) · Zbl 0562.73071 · doi:10.2514/3.9007

[7] Koeller, R.C.: Application of fractional calculus to the theory of viscoelasticity. ASME J. Appl. Mech. 51, 299–307 (1984) · Zbl 0544.73052 · doi:10.1115/1.3167616

[8] Metzeler, R., Nonnenmacher, T.F.: Fractional relaxation processes and fractional rheological models for the description of a class of viscoelastic materials. Int. J. Plast. 19, 941–959 (2003) · Zbl 1090.74673 · doi:10.1016/S0749-6419(02)00087-6

[9] Agrawal, O.P.: Application of fractional derivatives in thermal analysis of disk brakes. Nonlinear Dyn. 38, 191–206 (2004) · Zbl 1142.74302 · doi:10.1007/s11071-004-3755-7

[10] Deng, R., Davies, P., Bajaj, A.K.: A case study on the use of fractional derivatives: the low-frequency viscoelastic uni-directional behavior of polyurethane foam. Nonlinear Dyn. 38, 247–265 (2004) · Zbl 1142.74313 · doi:10.1007/s11071-004-3759-3

[11] Depollier, C., Fellah, Z.E.A., Fellah, M.: Propagation of transient acoustic waves in layered porous media: fractional equations for the scattering operators. Nonlinear Dyn. 38, 181–190 (2004) · Zbl 1099.74035 · doi:10.1007/s11071-004-3754-8

[12] Mainardi, F., Pagnini, G., Gorenflo, R.: Some aspects of fractional diffusion equations of single and distributed order. Appl. Math. Comput. 187, 295–305 (2007) · Zbl 1122.26004 · doi:10.1016/j.amc.2006.08.126

[13] Meerschaert, M.M., Benson, D.A., Scheffler, H.P., Baeumer, B.: Stochastic solution of space-time fractional diffusion equations. Phys. Rev. E 65, 1–4 (2002) · Zbl 1244.60080 · doi:10.1103/PhysRevE.65.041103

[14] Laskin, N.: Fractional market dynamics. Physica A 287, 482–492 (2000) · doi:10.1016/S0378-4371(00)00387-3

[15] Jumarie, G.: Fractionalization of the complex-valued Brownian motion of order n using Riemann–Liouville derivative: Applications to mathematical finance and stochastic mechanics. Chaos Solitons Fractals 28, 1285–1305 (2006) · Zbl 1099.60025 · doi:10.1016/j.chaos.2005.08.083

[16] Jumarie, G.: Path integral for the probability of the trajectories generated by fractional dynamics subject to Gaussian white noise. Appl. Math. Lett. 20, 846–852 (2007) · Zbl 1142.82013 · doi:10.1016/j.aml.2006.08.015

[17] Mainardi, F., Pironi, P.L.: The fractional Langevin equation: Brownian motion revisited. Extr. Math. 10, 140–154 (1996)

[18] Metzler, R., Klafter, J.: The random walk's guide to anomalous diffusion: a fractional dynamics approach. Phys. Rep. 339, 1–77 (2000) · Zbl 0984.82032 · doi:10.1016/S0370-1573(00)00070-3

[19] Agrawal, O.P.: Response of a diffusion-wave system subjected to deterministic and stochastic fields. Z. Angew. Math. Mech. 483, 265–274 (2003) · Zbl 1045.35116 · doi:10.1002/zamm.200310033

[20] Spanos, P.D., Zeldin, B.A.: Random vibration of systems with frequency dependent parameters or fractional derivatives. J. Eng. Mech. 123, 290–292 (1997) · doi:10.1061/(ASCE)0733-9399(1997)123:3(290)

[21] Rüdinger, F.: Tuned mass damper with fractional derivative damping. Eng. Struct. 28, 1774–1779 (2006) · doi:10.1016/j.engstruct.2006.01.006

[22] Agrawal, O.P.: Analytical solution for stochastic response of a fractionally damped beam. J. Vib. Acoust. 126, 561–566 (2004) · doi:10.1115/1.1805003

[23] Agrawal, O.P.: Stochastic analysis of dynamic systems containing fractional derivatives. J. Sound Vib. 247, 927–938 (2001) · doi:10.1006/jsvi.2001.3682

[24] Drozdov, A.D.: Fractional oscillator driven by a Gaussian noise. Physica A 376, 237–245 (2007) · doi:10.1016/j.physa.2006.10.060

[25] Ford, N.J., Simpson, A.C.: The approximate solution of fractional differential equations of order greater than 1. Numerical Analysis Report 386, Manchester Center for Numerical Computational Mathematics (2001) · Zbl 0976.65062

[26] Lin, Y.K., Cai, G.Q.: Probabilistic Structural Dynamics: Advanced Theory and Applications. McGraw-Hill, New York (1995)

[27] Seybold, H., Hilfer, R.: Numerical algorithm for calculating the generalized Mittag–Leffler function. SIAM J. Numer. Anal. 47, 69–88 (2008) · Zbl 1190.65033 · doi:10.1137/070700280

[28] Mainardi, F., Mura, A., Gorenflo, R., Stojanovic, M.: The two forms of fractional relaxation of distributed order. J. Vib. Control 13, 1249–1268 (2007) · Zbl 1165.26302 · doi:10.1177/1077546307077468

[29] Koh, C.G., Kelly, J.M.: Application of fractional derivatives to seismic analysis of base-isolated models. Earthq. Eng. Struct. Dyn. 19, 229–241 (1990) · doi:10.1002/eqe.4290190207

[30] Yuan, L.X., Agrawal, O.P.: A numerical scheme for dynamics systems containing fractional derivatives. In: Proceedings of 1998 ASME Design Engineering Technical Conferences, Atlanta, GA, Sept. 1998, pp. 13–16

[31] Shokooh, A., Suarez, L.: A comparison of numerical methods applied to a fractional model of damping materials. J. Vib. Control 5, 331–354 (1999) · doi:10.1177/107754639900500301

[32] Zhu, Z.Y., Li, G.G., Cheng, C.J.: A numerical method for fractional integral with application. Appl. Math. Mech. 24, 373–384 (2003) · Zbl 1142.74390 · doi:10.1007/BF02439616

[33] Huang, Z.L., Jin, X.L.: Response and stability of a SDOF strongly nonlinear stochastic system with light damping modeled by a fractional derivative. J. Sound. Vib. 319, 1121–1135 (2009) · doi:10.1016/j.jsv.2008.06.026

[34] Mainardi, F., Gorenflo, R.: On Mittag-Leffler-type functions in fractional evolution processes. J. Comput. Appl. Math. 118, 283–299 (2000) · Zbl 0970.45005 · doi:10.1016/S0377-0427(00)00294-6

[35] Suarez, L.E., Shokooh, A.: An eigenvector expansion method for the solution of motion containing fractional derivatives. J. Appl. Mech. 64, 629–635 (1997) · Zbl 0905.73022 · doi:10.1115/1.2788939