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Intersection form, laminations and currents on free groups. (English) Zbl 1242.20052
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Summary: Let F be a free group of rank $N \geq 2$, let μ be a geodesic current on F and let T be an \mathbb{R} -tree with a very small isometric action of F . We prove that the geometric intersection number $\langle T, \mu \rangle$ is equal to zero if and only if the support of μ is contained in the dual algebraic lamination $L^2(T)$ of T . Applying this result, we obtain a generalization of a theorem of *S. Francaviglia* [Trans. Am. Math. Soc. 361, No. 1, 161-176 (2009; Zbl 1166.20032)] regarding length spectrum compactness for currents with full support. We use the main result to obtain “unique ergodicity” type properties for the attracting and repelling fixed points of atoroidal iwip elements of $\text{Out}(F)$ when acting both on the compactified outer space and on the projectivized space of currents. We also show that the sum of the translation length functions of any two “sufficiently transverse” very small F -trees is bilipschitz equivalent to the translation length function of an interior point of the outer space. As another application, we define the notion of a filling element in F and prove that filling elements are “nearly generic” in F . We also apply our results to the notion of bounded translation equivalence in free groups.

MSC:

20F65 Geometric group theory
20E05 Free nonabelian groups
20F28 Automorphism groups of groups
20E08 Groups acting on trees
57M07 Topological methods in group theory
37A35 Entropy and other invariants, isomorphism, classification in ergodic theory
37B10 Symbolic dynamics
37E25 Dynamical systems involving maps of trees and graphs

Cited in **1** Review
Cited in **24** Documents

Keywords:

free groups; outer space; geodesic currents; geometric intersection numbers; isometric actions; translation length functions; algebraic laminations; filling elements

Full Text: [DOI](#) [arXiv](#)

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