

Artebani, Michela; Dolgachev, Igor**The Hesse pencil of plane cubic curves.** (English) Zbl 1192.14024

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This paper deals with a well-known configuration of 9 points and 12 lines in $\mathbb{P}^2(k)$, the Hesse configuration, in which each point lies on 4 lines and each line contains 3 points. Such points can be chosen as the nine inflection points of a nonsingular plane cubic curve, and they can be as well taken as common inflection points of the Hesse pencil

$$\lambda(x^3 + y^3 + z^3) + \mu xyz = 0.$$

The group of plane automorphisms preserving the Hesse pencil has order 216 and it is isomorphic to $(\mathbb{Z}/3\mathbb{Z})SL(2, \mathbb{F}_3) \rtimes SL(2, \mathbb{F}_3)$. The algebra of invariant polynomials of one of its extensions to a subgroup of $GL(3, \mathbb{C})$ has a generator of degree 6 defining a plane sextic C_6 .

The double covers of \mathbb{P}^2 branched over C_6 and over the singular sextic C'_6 with 8 cuspidal singularities are both $K3$ surfaces and they are singular in the sense of Shioda, i.e. the subgroup of algebraic cycles in the second cohomology group is of maximal rank.

The authors compute the intersection form defined by the cup-product on these subgroups and describe the geometrical meaning of the set of intersection points C_6 cuts each curve of the Hesse pencil at.

Reviewer: [Eleonora Palmieri \(Roma\)](#)**MSC:**

- [14H10](#) Families, moduli of curves (algebraic)
- [14H50](#) Plane and space curves
- [14J10](#) Families, moduli, classification: algebraic theory

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configuration; pencil; double covers

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