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Analysis of plates and shells using an edge-based smoothed finite element method. (English)

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The authors present an approach to the analysis of arbitrary thin to moderately thick plates and shells by an edge-based smoothed finite element method (ES-FEM). The formulation is based on the first-order shear deformation shell theory. Triangular meshes are used. The discretized systems of equations are obtained using the smoothed Galerkin weak form, and the numerical integration is applied based on the edge-based smoothing domains. The development of effective triangular elements for plates and shells is not a trivial matter. The major difficulties are to overcome the overly stiff behavior and the shear locking phenomenon. The simple change of smoothing domains gives the ES-FEM excellent properties including good accuracy and the absence of spurious modes. In this work, the ES-FEM has been further extended to solve plates and shells. The smoothing domains are associated with the edges of the triangles. Compared with FEM, the smoothing operation reduces the stiffness of the discretized system, and compensates nicely the overly stiff behavior of the FEM model. The ES-FEM can produce better solutions than the corresponding FEM model. To validate the accuracy and stability of the ES-FEM, a number of numerical examples have been examined, and comparisons are made with results available in the literature. Good results have been obtained for both plate and shell problems.

Reviewer: [V. Leontiev \(Ul'yanovsk\)](#)

MSC:

[74S05](#) Finite element methods applied to problems in solid mechanics

[74K20](#) Plates

[74K25](#) Shells

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