Levin, David A.; **Luczak, Malwina J.**; **Peres, Yuval**

Glauber dynamics for the mean-field Ising model: cut-off, critical power law, and metastability. (English) Zbl 1187.82076

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One considers Glauber dynamics for the Ising model on sequences of transitive graphs. It is shown that the system exhibits a cut-off for values of the absolute temperature T larger than the unity. When $T = 1$, one can obtain the order $n^{3/2}$ of the mixing time, and the meta-stability of the system is analyzed when T is small. In this case, it is shown that the mixing time is logarithmic

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MSC:

82C20 Dynamic lattice systems (kinetic Ising, etc.) and systems on graphs in time-dependent statistical mechanics
60J10 Markov chains (discrete-time Markov processes on discrete state spaces)
60K35 Interacting random processes; statistical mechanics type models; percolation theory

Keywords:

Ising model; Glauber dynamics; Markov chains; Curie-Weiss model; mixing time; cut-off; coupling; meta-stability; heat-bath dynamics; mean-field model

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