Acerbi, Emilio; **Fusco, Nicola**

Semicontinuity problems in the calculus of variations. (English) Zbl 0565.49010

Arch. Ration. Mech. Anal. 86, 125-145 (1984).

The authors give some semicontinuity and relaxation results for integrals of the calculus of variations. The following is one of the most interesting theorems proved in the paper. It is a very deep result: Let Ω be an open set in $\mathbb{R}^n$. Let us assume that $f = f(x, s, \xi)$ is a real Carathéodory function defined in $\Omega \times \mathbb{R}^m \times \mathbb{R}^{nm}$, quasiconvex with respect to ξ in Morrey's sense, and such that $0 \leq f(x, s, \xi) \leq a(x) + c(|s|^p + |\xi|^p)$ for a.e. $x \in \Omega$, and for every $s \in \mathbb{R}^m$, $\xi \in \mathbb{R}^{nm}$, where c is a positive constant, $p \geq 1$ and $a \in L^1_{loc}(\Omega)$. Then the functional

$$u \in W^{1,p}(\Omega; \mathbb{R}^m) \to \int_\Omega f(x, u(x), Du(x)) \quad dx$$

is sequentially lower semicontinuous in the weak topology of $W^{1,p}(\Omega; \mathbb{R}^m)$. A result of existence of minima by the reviewer and *C. Sbordone* [J. Math. Pures Appl. 62, 1-9 (1983; Zbl 0516.49011)] and a semicontinuity theorem by the reviewer [Manuscripta Math. 51, 1-28 (1985)] are related to the quoted semicontinuity theorem. Moreover, the book by *B. Dacorogna* ["Weak continuity and weak lower semi-continuity of non-linear functionals", Lect. Notes Math. 922 (1982; Zbl 0484.46041)] is related to the relaxation results.

Reviewer: P.Marcellini

MSC:

49J45	Methods involving semicontinuity and convergence; relaxation
26B25	Convexity of real functions of several variables, generalizations
46E35	Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems
54C08	Weak and generalized continuity
49J10	Existence theories for free problems in two or more independent variables

Keywords:

semicontinuity; relaxation

Full Text: DOI

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