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On a diffuse interface model for two-phase flows of viscous, incompressible fluids with matched densities. (English) Zbl 1254.76158

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Summary: We study a diffuse interface model for the flow of two viscous incompressible Newtonian fluids of the same density in a bounded domain. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed in the model. Moreover, diffusion of both components is taken into account. This leads to a coupled Navier-Stokes/Cahn-Hilliard system, which is capable of describing the evolution of droplet formation and collision during the flow. We prove the existence of weak solutions of the non-stationary system in two and three space dimensions for a class of physical relevant and singular-free energy densities, which ensures – in contrast to the usual case of a smooth free energy density – that the concentration stays in the physical reasonable interval. Furthermore, we find that unique "strong" solutions exist in two dimensions globally in time and in three dimensions locally in time. Moreover, we show that for any weak solution the concentration is uniformly continuous in space and time. Because of this regularity, we are able to show that any weak solution becomes regular for large times and converges as $t \to \infty$ to a solution of the stationary system. These results are based on a regularity theory for the Cahn-Hilliard equation with convection and singular potentials in spaces of fractional time regularity as well as on maximal regularity of a Stokes system with variable viscosity and forces in $L^2(0,\infty;H^s(\Omega))$, $s\in[0,\frac{1}{2})$, which are new themselves.

MSC:

76T99 Multiphase and multicomponent flows
35Q30 Navier-Stokes equations

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Navier-Stokes system; existence; regularity; weak solution; Cahn-Hillard equation

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