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Variational principles and free-boundary problems. (English) Zbl 0564.49002

Pure and Applied Mathematics. A Wiley-Interscience Publication. New York: John Wiley & Sons, Inc. IX, 710 p. (1982).

Let J be a convex function, bounded below, from a Banach space X into $\mathbb{R}$ and let $\mathcal{K}$ be a closed convex set in X. Consider the problem of minimizing $J(x)$, $x \in \mathcal{K}$. If J is differentiable in $\mathcal{K}$, the inequality that characterizes the solution u, whenever it exists, i.e., $u \in \mathcal{K}$, $< J'(u), v - u >\geq 0$ for all $v \in \mathcal{K}$ where $< \cdot, \cdot >$ is the duality between X and X', is referred to as a variational inequality.

Problems of this kind have been widely studied over the past fifteen years. A large part of their fortune is due to the fact that a number of problems from physics can be mathematically reformulated in terms of variational inequalities. Several of these problems are described, as a common feature, by a partial differential equation in some (unknown) domain on whose boundary some extra conditions are prescribed. They are called free boundary problems. In the latter situations, beside questions of existence and uniqueness, the properties of the solutions are relevant, chiefly the structure and smoothness of the free boundary. The aim of the book is to give an account of the main results established in the past ten years in this direction.

As a framework, a spectrum of physical problems is considered; we mention specifically the stretching of a membrane over an obstacle, stationary filtration in a porous dam, elasto-plastic material under stress, and behavior of fluid jets out of a nozzle. The problems are first reformulated in variational form, existence and uniqueness is established, and their properties are investigated.

The soft analysis is limited to the bare minimum necessary to prove existence and uniqueness. Clearly the main concern of the book is to derive information on the solution and especially the free boundary. This is usually the result of hard classical analysis. Most of the techniques have been designed ad hoc for particular free boundary problems.

At first glance the collection of problems may seem to be disconnected, notwithstanding their common variational structure. In fact they illustrate the main new methods in analysis, generated by the study of free boundary problems. Unfortunately no mention is made of numerical methods. The last chapter contains a study of some nonvariational problems such as time-dependent diffusion of a gas in a porous medium and n-dimensional two-phase Stefan problem.

The book is well written and the style is clear. It is also quite accessible given some background in classical partial differential equations. Here and there some details are left to the reader. A few misprints are present.

Because of the large amount of material collected in it, the reviewer finds the book a valuable reference text for free boundary problems with variational structure.

MSC:

49J40	Variational inequalities
49-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to calculus of variations and optimal control
35R35	Free boundary problems for PDEs
35Q99	Partial differential equations of mathematical physics and other areas of application
35J85	Unilateral problems; variational inequalities (elliptic type) (MSC2000)
74S30	Other numerical methods in solid mechanics (MSC2010)
74C99	Plastic materials, materials of stress-rate and internal-variable type
74K15	Membranes
76D99	Incompressible viscous fluids
76S05	Flows in porous media; filtration; seepage

Keywords:

variational inequalities; free boundary problems; structure and smoothness of the free boundary; stretching of a membrane over an obstacle; stationary filtration in a porous dam; elasto-plastic material under stress; behavior of fluid jets out of a nozzle; two-phase Stefan problem